



DISSEMINATING INQUIRY-BASED SCIENCE  
AND MATHEMATICS EDUCATION IN EUROPE

# DYNAMIC WORKSHEETS: A TOOL FOR PROBLEM- BASED INDIVIDUAL AND COOPERATIVE LEARNING

WITH THE SUPPORT OF



# Dynamic Worksheets: A Tool for Problem-Based Individual and Cooperative Learning

The teachers' role should not be to lecture but to guide students on journeys on discovery.

Learning is an individual process that can be suggested and influenced from outside, but the real learning processes go on in the interior of every single person. Therefore the possibilities of an external control are known to be effective only in a limited way.

If learning is to be successful the students must be able to tie their own think-nets. The more connections they establish between elements of knowledge, i.e. the denser and tighter the nets are woven, the more flexible thinking becomes. Thus the students are more likely in a position to apply their knowledge in changed or unfamiliar situations. This means that meaningful learning is active learning by discovery. We have to say good bye to the dominant lecturing lessons.

Dynamic worksheets and learning environments enable first successful steps towards active problem-based learning by discovery. In comparison to a traditional lesson the teacher does not mainly give his own knowledge to the students, but he helps them to achieve an individual access to the subject matter.

## Experiments on the computer screen – records in a study journal

Teaching takes place under a problem-oriented aspect. We start with specific activities on the computer screen. The students are asked to do some experiments by themselves. The aim is to discover certain properties or relationships and to write down assumptions. Mathematics turns out to be an experimental science. The PC is the laboratory; a dynamic configuration on the screen represents the experiment. To get sustainability the experiments together with the results have to be recorded in a study journal. We prefer the old-fashioned writing and sketching with a pencil on paper. This method perfectly complements the working with the electronic dynamic worksheets.

The students use their study journal for

- ▶ sketching meaningful figures,
- ▶ describing their observations,
- ▶ phrasing assumptions,
- ▶ writing down proofs,
- ▶ expressing individual impressions and comments.

While exploring dynamic worksheets students often are asked to take notes into their study journals. These individual records are used as basis for an overview of the discussed subject matter. For some learning environments so-called result-sheets exist as PDF-file. They contain a summary of the unit and complete the individual records of the students.

## School is more than a place for lecturing – school is a place for learning

After studying alone a dynamic worksheet the students are asked to exchange their ideas and results, to compare and to complete them. Here they practice orally and in writing how to express themselves mathematically. This active discussion leads to a deeper understanding of the mathematical topic.

The part of the teacher changes, he more and more becomes an observer. He advises, he assists his students, he gives them help through self-help, he notices which solutions they have worked out and he chooses students for presentation. Working with dynamic worksheets shows that learning is not a passive process.

The teacher chairs the presentation and the discussion of the students' results. Afterwards – in a teacher-centred stage - he gives a summary and connects the new results to prior mathematical knowledge. Furthermore he may show standard proofs and introduces new mathematical terms, if necessary.

Reasons for using dynamic worksheets:

- ▶ Students have to be active.
- ▶ Promoting independent working.
- ▶ Students are largely able to decide their own learning speed.
- ▶ The experimental access arouses their interest.
- ▶ Moving and changing of configurations leads to new insights.
- ▶ Dynamic visualization supports the understanding of geometrical properties.
- ▶ No previous technical knowledge is necessary.
- ▶ No familiarity with mathematical software is needed.

## Dynamic worksheets – geometry is alive

In our learning arrangement with dynamic worksheets the students have to go through learning processes in three stages:

- ▶ alone,
- ▶ together with other students,
- ▶ discussing with the whole class and/or the teacher.

My Swiss colleagues Peter Gallin and Urs Ruf have briefly and to the point denoted these stages with the triple step I – You – We. The following learning environment *Theorem of Thales* is strictly modelled on this structure. Goals and activities of the three stages are described below. By *Theorem of Thales* we understand a standard result of triangle geometry:

[AB] is a diameter of a circle and C is any other point on the circle.  
Then triangle ABC is a right triangle with  $90^\circ$  angle at point C.

## Stage 1

The learning environment starts with a problem that stimulates the students to explore the chosen topic. Each worksheet contains a dynamic configuration and short instructions that lead to own experiments, observations and discoveries. The students are asked to draw sketches in their study journal, to describe their observations and to phrase conjectures. By doing this they learn to structure their thoughts and to express them in writing. These notes are the basis for the following discussion of the results.

The worksheet itself has a traditional textbook design. But in contrast to a textbook the students have the possibility to experiment.

What does this mean? They drag vertex C, they observe the interior angles of the triangle and they change the diameter. Short instructions stimulate the activities. Finally the students are asked to formulate a conjecture that is to be written down in the study journal.

### Theorem of Thales

The endpoints A and B of a diameter are connected with an arbitrary point C on the circle.

$\alpha = 26.77^\circ$   
 $\beta = 63.23^\circ$   
 $\gamma = 90.0^\circ$

- Move C on the circle.
- Observe the size of the interior angles of triangle ABC.
- What do you assume? Take notes.

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## Stage 2

Afterwards the students compare their results with their neighbours and complete their notes in the study journal, if necessary. Only now they are having a look at the next worksheet. Here they find

### Theorem of Thales

Compare with your assumption:

If the vertices of a triangle lie on a circle with one side as diameter, then the triangle has a right angle.

This result is often attributed to Thales of Milet (ca. 620 - ca. 540 v. Chr.). But this connection is extremely uncertain, because we have no reliable historical information. Nevertheless the name Theorem of Thales is used especially in German speaking countries.

The proof follows the strategy:

Use a [convenient auxiliary line](#).

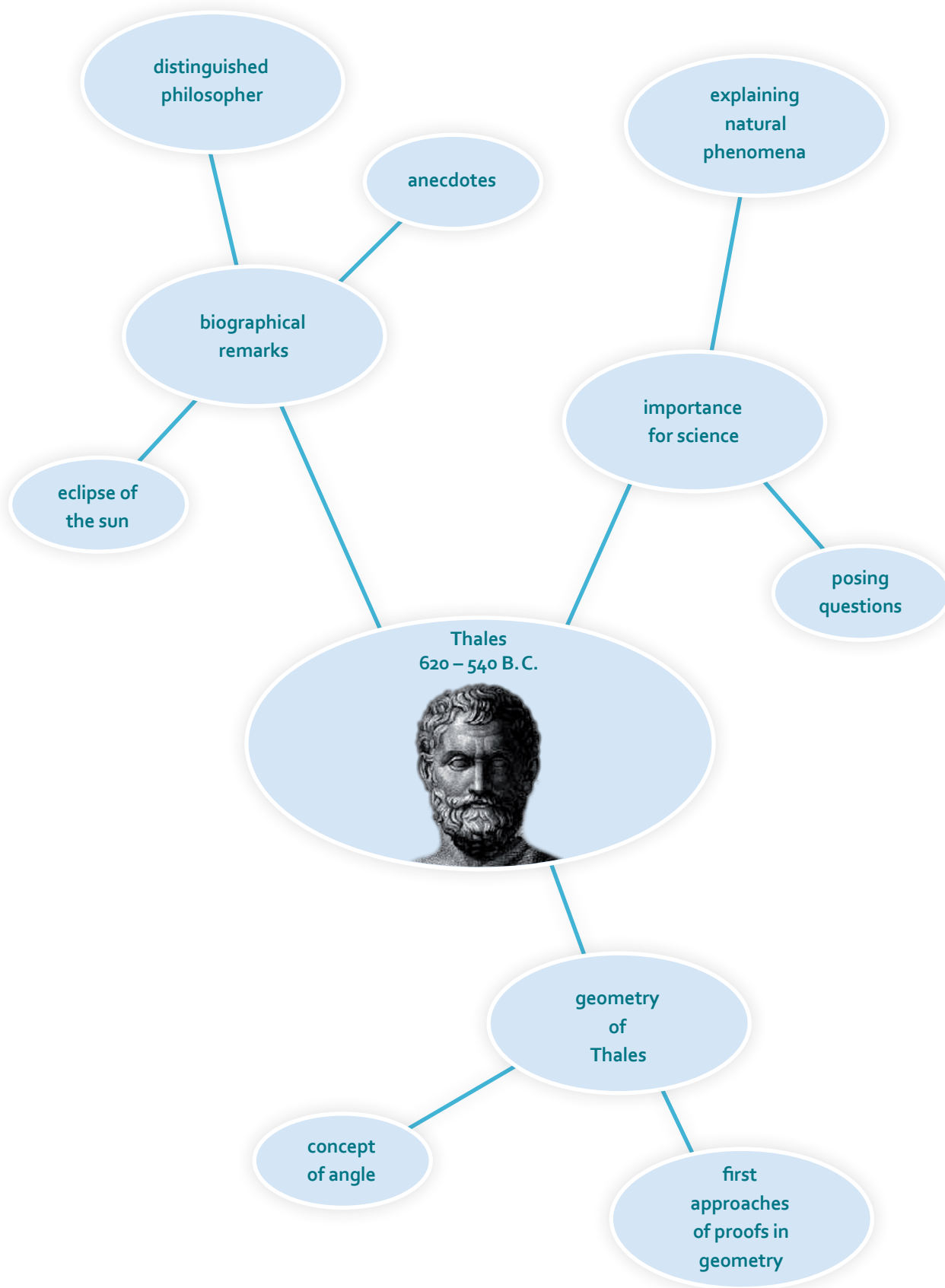
To do this, connect the vertex C with center M of the side [AB].

[Hint](#)

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hints how to prove the assumptions and the suspected properties. Under this guideline the students first try to take their own steps towards a proof. Ideally the teacher has only the part of an adviser.

Apart from the Theorem we find a historical remark here. The teacher can take this opportunity to tell some biographical details and stress the importance of the philosopher Thales for the development of science. The following diagram gives an overview on various topics in this context:



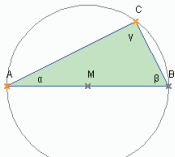
## Stage 3

This stage is similar to the traditional way of teaching. First the teacher picks up results of the students or the students themselves show their own proofs. The corresponding worksheet shows

a standard proof of the *Theorem of Thales*. For better illustration we choose a presentation of the proof with so-called mouse-over-effects. So it is easier for the students to recognise specific details in the configuration.

**Theorem of Thales**

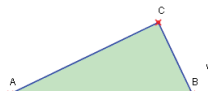
If the vertices of a triangle lie on a circle with one side as diameter, then the triangle has a right angle.



$\beta = \angle CBA = \angle MCB$  ( $\triangle BCM$  isosceles)  
 $\alpha = \angle BAC = \angle ACM$  ( $\triangle ACM$  isosceles)  
 $\rightarrow \gamma = \alpha + \beta$   
 $180^\circ = \alpha + \beta + \gamma =$   
 $\rightarrow \gamma = 90^\circ$

Find the converse of the [Theorem of Thales](#)!

GEONEXT



Move point C so that the angle  $\gamma$  has always  $90^\circ$ .

[Show the Thales circle](#) and compare with the trace you have drawn.

This worksheet can also be used in a teacher-centred lesson. On the other hand students can work on their own with it and the teacher checks their understanding with suitable questions.

Often teachers need an additional worksheet for smarter students while the rest of the class is still working with the above sheets. We suggest an experiment that leads to the converse of the *Theorem of Thales*.

## Conclusion

The advantage of our triple step method is that the students have to study three times the same problem but each time under a different point of view:

- 1) Individual work.
- 2) Cooperation with a partner or a small learning group.
- 3) Communication with the whole class and additional teacher's instructions.

So we have no boring repetition and therefore students get a deeper understanding of the problem and the solution process. As they learn in pairs or in groups their learning experiences are not isolated or individual. Thus the students improve their competency in communication and cooperation. We do not use technology in the classroom for technology reason. Our major goal is to improve learning and mathematical understanding. In conclusion the following diagram shows which different methods interact while working with dynamic worksheets.



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