The Fibonacci Project

DISSEMINATING INQUIRY-BASED SCIENCE AND MATHEMATICS EDUCATION IN EUROPE

RUTH DOLENC-PETZ, PETRA IHN-HUBER (ED.)

SUPPORTING GEOMETRIC COMPETENCES

WITH THE SUPPORT OF

EUROPEAN COMMISION

SEVENTH FRAMEWORK PROGRAMME

Ruth Dolenc-Petz, Petra Ihn-Huber (Ed.)

Supporting Geometric Competences

DISSEMINATING INQUIRY-BASED SCIENCE
AND MATHEMATICS EDUCATION IN EUROPE

The CD-ROM provides

- worksheets for the pupils and
- background information for the teacher

as Microsoft® Word® documents and PDF files.

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The Fibonacci project deals with disseminating inquiry-based science and mathematics education in Europe. Within Europe, strategies in order to develop methods of teaching mathematics and science and concepts for teachers' professional development are designed, tested, evaluated and implemented. For further information on the Fibonacci project visit: http://fibonacci-project.eu

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Contents

Introduction

Improving Knowledge – Explaining Phenomena

"He who understands geometry is able to understand everything in this world."
Galileo Galilei

Educational contents for geometry are still addressed more or less intensively in primary school and, regrettably, play all too often an episodic part. A continuous supply of geometrical contents is necessary for various reasons.

- Geometry lessons support the development of cognitive skills, the development of general intelligence, and the capacity to think and work in three dimensions.
- Geometry lessons contribute to the students' perception of the environment (three-dimensional orientation, spatial affiliation, geometrical analysis of shapes and spaces, measurement of lengths and surface area).
- Geometry lessons support conceptualization (terms for three-dimensional connections, geometrical terms for shapes, terms for geometric connections) and therefore cognitive development, brain capacity and problem solving.

In the context of specialized tasks in math classes, geometric thinking often proves to be the base for the imagining of the arithmetic area, e.g. in the use of three-dimensional materials, illustrations and graphics. Without geometric thinking, it is also difficult to develop reasonable concepts in arithmetic.

Moreover, through geometry, it is possible to convey a positive attitude towards the subject mathematics. Activity-oriented and creatively-formatted tasks that offer the opportunity to experience the beauty of mathematics beyond arithmetical tasks motivate virtually every student.

In addition, children, especially those who find calculating difficult, often get a feeling of success in studying geometry. This is experienced frequently at preschool age and can be later tied to feelings of success at school. Thus, success in class is easier to achieve, and the children's efforts to learn pay off visibly.

The substantial number of tasks in this book are meant to open up to the diverse possibilities of planning a geometry class and give tangible suggestions for developing three dimensional imagination and acquiring geometrical knowledge and competency in geometric drawing as basic parts of a geometry class. The tasks in this book are based on the framework of topics which is given in the standards of education for primary schools, namely:

- terminology for geometric qualities,
- three-dimensional connections,
- plane figures and shapes,
- solid forms,
- properties of symmetry,
- geometric drawing,
- mappings and movements,
- nets and paths,
- lines and segments.

These topics are illustrated in the areas of three-dimensional orientation, solid forms, plane figures and symmetries. The practical and objective involvement of the learning content is also of significance.

The acquisition of expertise can only happen through multiple encounters with selected tasks. To enable an intensive involvement with a subject, we have allocated several worksheets at different levels to each of the individual chapters.

Aside from the encouragement of content-related competence, the tasks in the book aim to initiate and encourage general mathematical competence in skills like arguing, depicting, communicating, modeling and problem solving.

We wish you success in sparking the children's interest and enjoyment in "doing geometry" with the help of our proposed activities.

Guidelines for Using the Worksheets

"Sustainable learning, memorizing and interlinking knowledge is eased and supported by action, visual perception and language."
Marianne Franke

This book contains worksheets and provides detailed guidance in the areas three-dimensional orientation, solid forms, plane figures and symmetry.

The tasks within each chapter are grouped thematically.

Each section contains reference to standards of education, brief didactic commentary and methodological advice for practical use in class. Five or six worksheets are allocated to each section.

The worksheets are numbered consecutively.

For primary school children it is essential to have the opportunity to achieve knowledge by experiencing activities. Therefore suggestions are consistently given in the methodological advice for an activity-oriented introduction of each topic.

The tasks are designed to enable each learner to acquire the basic skills. Additionally, further options are given for high-performing children.

1 At the Zoo

Three-dimensional Orientation – Mental Geometry in Planes

E. Kick

Reference to Standards and Competences

Spaces and Shapes:

- Recognizing, describing and using three-dimensional relationships (paths, maps/plans, perspectives), developing spatial awareness

Background and Didactic Commentary

For many people, it is natural to perceive their three-dimensional environment through their eyes. Therefore visual perception is considered to be a basic skill, without which three-dimensional imagination and thinking cannot be developed.

Visual perception is of great significance in the areas of reading and writing as well as in arithmetic (e.g. interpreting reversed letters and numerals).
Prior to solving geometrical tasks mentally and at abstract levels, pupils need develop their visual perception by working in concrete terms with given images.

Methodological Advice

Exercises such as those that follow in visual perception are appropriate at a very early stage, such as the first grade. As a starting point for all the tasks we have chosen the context of a "zoo", an area that relates to many children's everyday reality.
Task 1 in both Worksheets (WS) 1 and 3 is about recognizing figures and differentiating between figures. WS 2 deals with identifying specific figures from a complex background. Here concentrating on the important details is crucial.
Mazes are central to Task 2 in both WS 1 and 3. In order to encourage pupils to follow the paths with their eyes and not their hands, it is recommended to ask pupils to keep both hands behind their backs.
In WS 4, with the aid of a token, pupils can follow the directions for the map of the zoo as read by the teacher or a partner. Furthermore, the children can create and outline their own map or directions. This plan could serve as a base for the communication with a partner (see general mathematical competences: outlining and communicating).
WS 5 is about the perception of positions in a space and the differentiation right-left.

Name: Date:

WS 1

At the Zoo

1. There are many visitors at the Zoo “Eagle’s Eye“.
   Match the silhouettes 1 - 4 with the right persons.




Hans	Rudi	Otto	Lisa

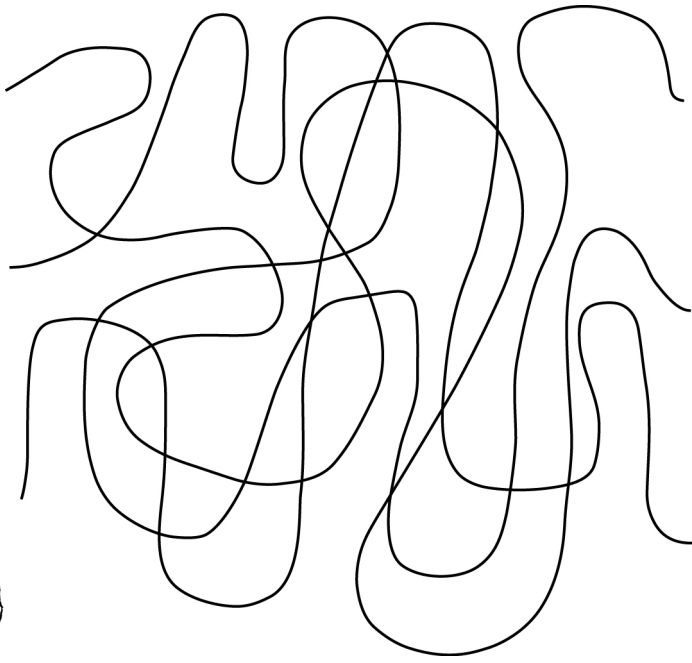


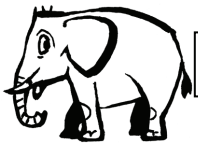

1	2	3	4

Hans: _____ Rudi: _____ Otto: _____ Lisa: _____

2. Each of the three children has a favorite animal.
   First follow the paths with your eyes only.
   Then trace the paths with different colors and write down.






giraffe

elephant

zebra

Hans: _____ Rudi: _____ Lisa: _____

Name: Date:

WS 2

Animals at the Zoo

1. Can you find the camel? Can you find the monkey? Color the animals.

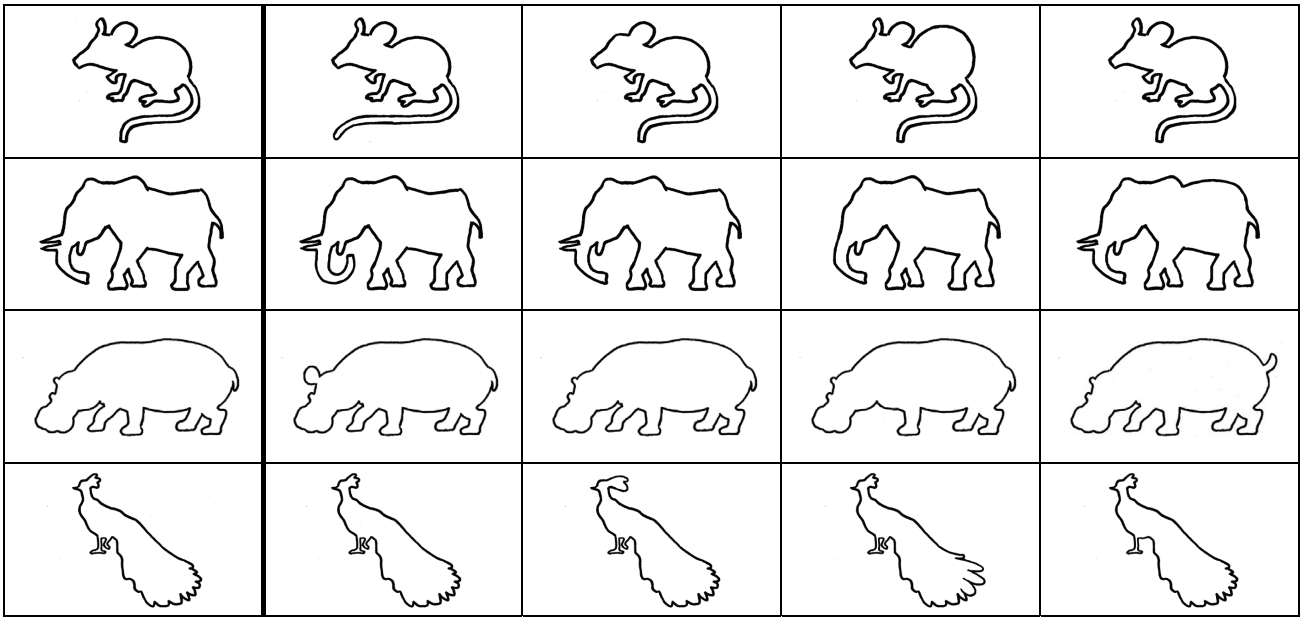
2. Find the four animals in the picture and color them.

Name: Date:

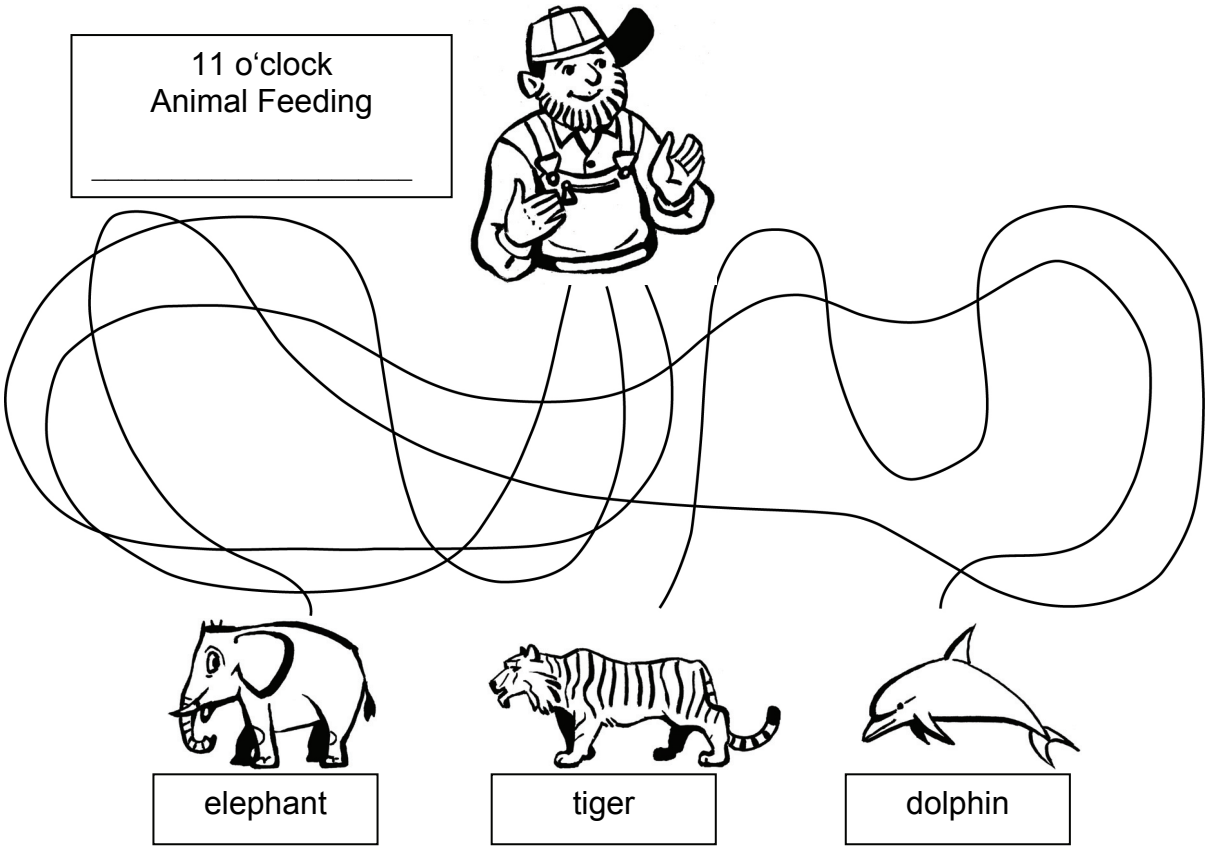
WS 3

Animal Feeding

1. Which animal is identical with the left one? Color it.



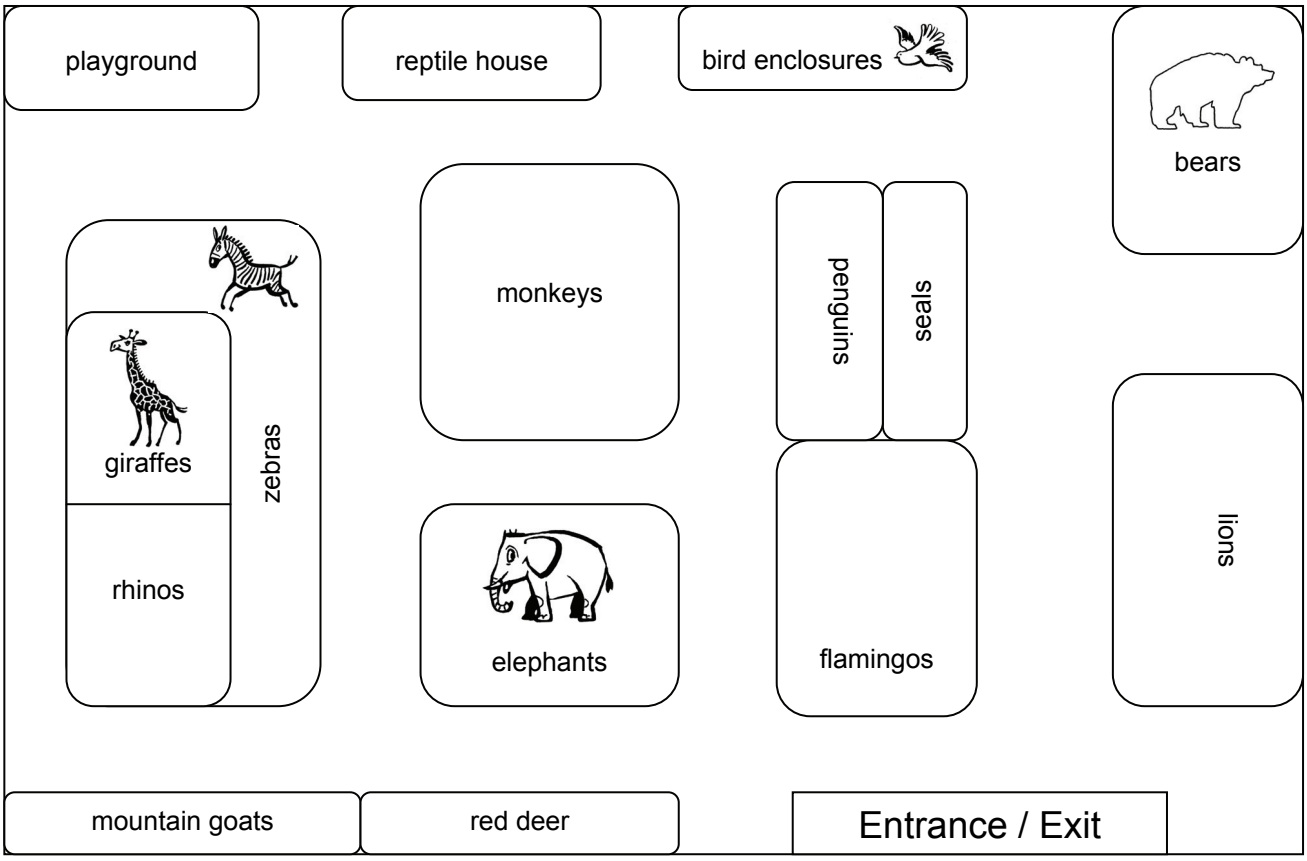
2. At 11 o'clock the visitors can watch an animal feeding.
   Which animal do you think it is? Write it on the board.
   First follow the path with your eyes only. Then check with a colored pencil.



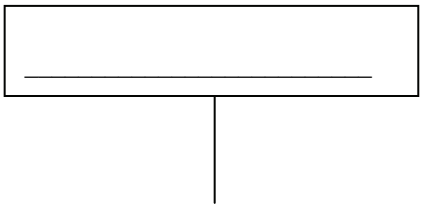
Name: Date:

WS 4

Overview of the Zoo “Eagle’s Eye“



1. You start at the entrance. You turn to the left.
   You walk straight ahead, past the elephants.
   Then you go to the right.
   In front of the animals on the left side you stop.

   On the board you read: 

2. Which animals can you find to the right of the bears? _____

3. You are in front of the seals. Describe two possibilities to get to the monkeys.

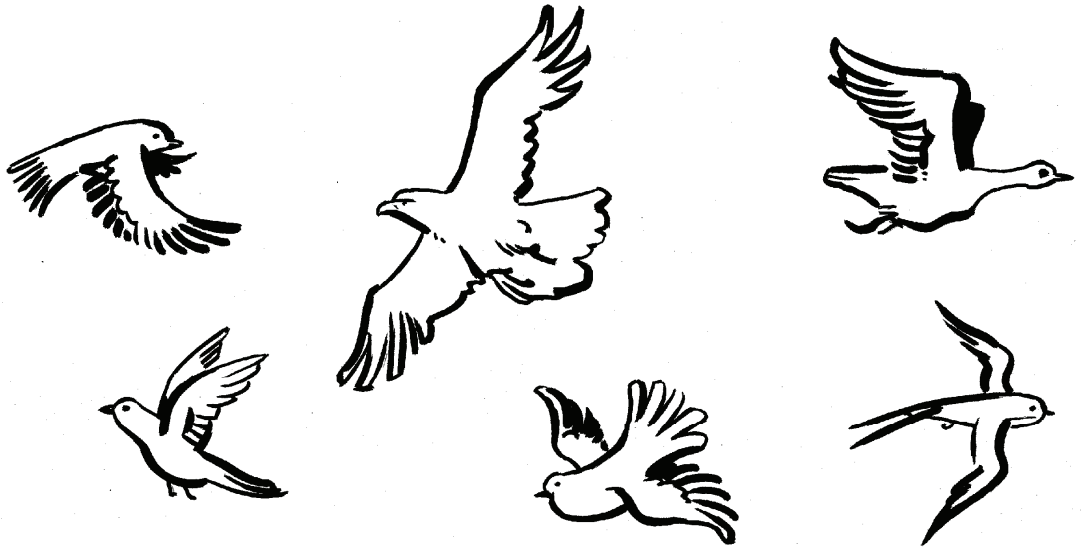
4. You are in front of the giraffes. Which is the shortest way back to the exit?

5. Draw a route through the zoo. Describe this route to your partner. Your partner must sketch your route according to your description on his / her map of the zoo. Compare.

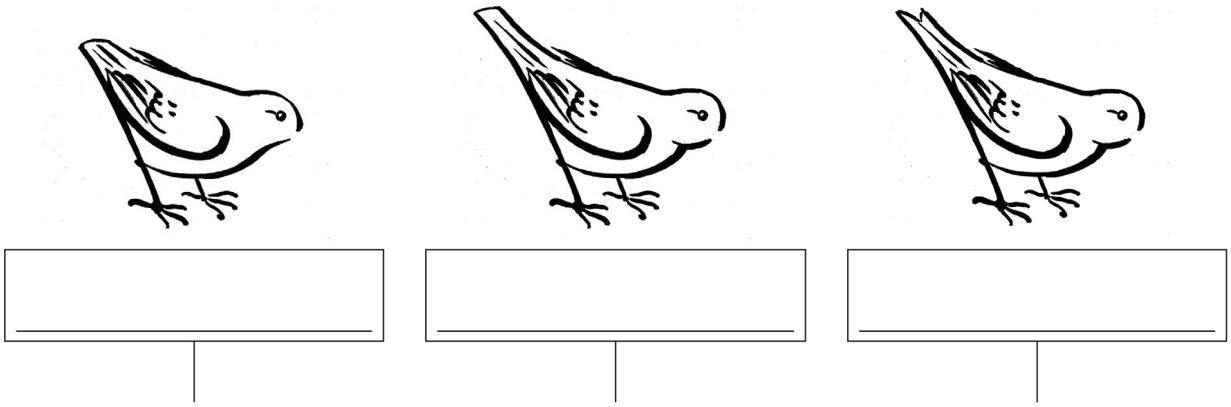
Name: Date: WS 5

In the Bird Enclosure

1. Left – Right?
   Circle all the birds that fly to the right in red,
   and all the birds that fly to the left in blue.



2. Follow the directions given below to draw the beak on each bird
   and write down the appropriate name for each bird.



- Each bird has a sharp or a curved beak.
- Each bird has a yellow or a brown beak.
- The nuthatch doesn't have a yellow or a curved beak.
- The bird on the right has a pointed beak.
- The bird in the middle is the only one with a yellow beak.
- The treecreeper is sitting the farthest away from the bird with the brown, sharp beak.
- The bird next to the only bird with the bent beak is the starling.

2 Visual Travel

Three-dimensional Orientation – Mental Geometry in Spaces

R. Dolenc-Petz

Reference to Standards and Competences

Spaces and Shapes:

- Developing spatial awareness; orienting oneself in spaces; connecting two- and three-dimensional graphics; recognizing, naming and depicting geometric figures

Background and Didactic Commentary

Developing our three-dimensional imagination and thinking enables us to orient ourselves in the environment. To master our daily routine, it is inevitable to imagine objects, relationships between objects and processes in a three-dimensional way. Developing the competence to imagine three-dimensional objects and processes must be continuously encouraged, particularly as the development stages of the spatial geometrical thinking are evolved very differently.

The reconstruction and naming of features of solid forms from our imagination deepens the existing basic knowledge of solid forms, supports the three-dimensional imagination and at the same time presents information of a more or less distinctive (figural) memory. Rotating and flipping geometric forms and their compositions mentally as well as describing them from imagination requires and encourages three-dimensional thinking. For this purpose, it is important to have children move solid forms or their compositions mentally into other positions and then imagine the connections between the forms in the altered position.

Methodological Advice

A basic knowledge of the attributes of three-dimensional forms is required to successfully solve the tasks in this chapter. All the activities in WS 6-8 can be started and supported using the physical three-dimensional forms (cube, cuboid, sphere, cone, cylinder, pyramid, triangular prism; note that the latter is simply called “prism” in this book). The tasks have been designed to support experimental actions and testing as well as mental activities and verbalizing. Especially children with less-developed geometric thinking should approach the solutions of the tasks actively, and only after this active approach will they comprehend the tasks mentally.

Task 3 in WS 6 allows various interpretations that the children can imagine in communication with partners. When working with the riddle cards in WS 7, the pupils have to remember the different features of the geometric forms. The sketch requested at the end of each riddle card serves as a tool to identify the pupils’ competencies or lack thereof in three-dimensional drawing. At this early stage, the teacher should be content with modest results (when compared with the section regarding three-dimensional drawing).

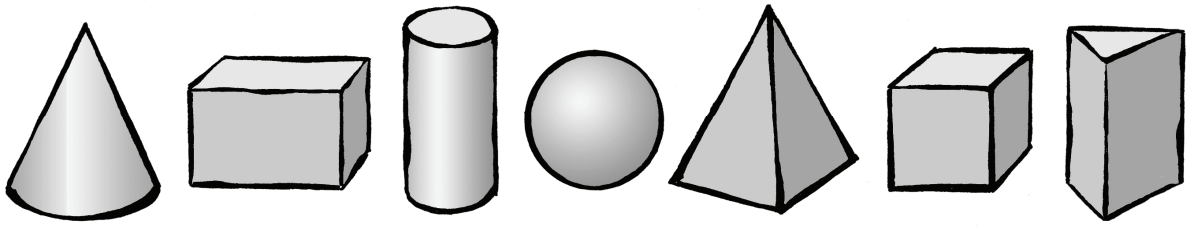
To imagine three-dimensional movements like rotating and flipping the forms, the tasks in WS 9 and 10 can help. The given objects can be made with cubes for activity-oriented processing (or the object may also be provided ready-made). The tasks can be solved by rotating the forms or building the missing parts. In WS 10 the first two tasks are easier to manage, because the complements don’t have to be turned. With the help of physical materials, the children can invent similar tasks themselves, which their partners can then solve.

Name: Date:

WS 6

Solid Forms and Buildings

1. Connect the drawings of the three-dimensional forms with the correct names.

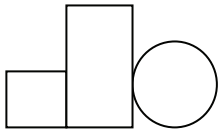


sphere	prism	cuboid	cone	cylinder	cube	pyramid

2. Complete the correct number.

	number of vertices	number of edges	number of faces
cuboid	_____	_____	_____
cube	_____	_____	_____
pyramid	_____	_____	_____
cylinder	_____	_____	_____
sphere	_____	_____	_____
prism	_____	_____	_____

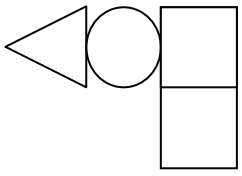
3. The following buildings are seen from above.
   Write down which forms have been used for each building.
   Discuss with your partner.



forms: _____

my explanation: _____

_____



forms: _____

my explanation: _____

_____

Name: Date:

WS 7

Riddles with Solid Forms

Write on the riddle cards which forms have been described. Sketch them.

I have eight vertices and twelve edges. My opposite faces are rectangles of equal size.

my name: _____

sketch:

I consist of a circular face and a curved side.

my name: _____

sketch:

I have a quadratic base. My faces are triangles of equal size.

my name: _____

sketch:

I have three faces: a rectangle and two circular faces of equal size.

my name: _____

sketch:

I have six quadratic faces of the same size, twelve edges and eight vertices.

my name: _____

sketch:

I have neither edges nor vertices.

my name: _____

sketch:

Name: Date:

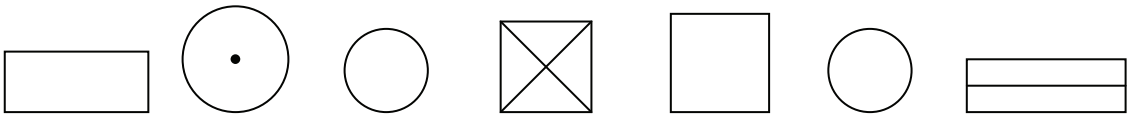
WS 8

A Matter of View

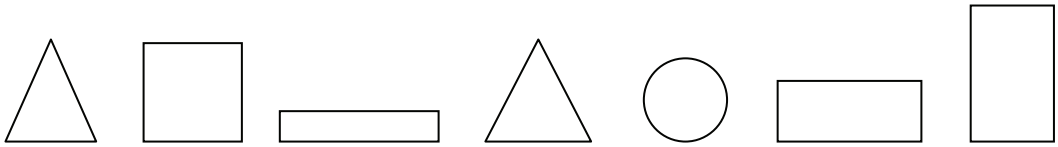
1. Mark the name of the form and its matching views using the same color.

cube	sphere	cuboid	prism	cylinder	cone	pyramid

View from above



View from front



2. Tick which features apply to the different forms.

	cube	pyramid	cylinder	cone	cuboid	sphere
The form has 6 faces.						
The form has 4 faces at the most.						
At least 2 faces are quadratic.						
All faces are quadratic.						
All faces are rectangular.						
The form has at least 8 edges.						
The form has 12 edges.						
The form has at least 6 vertices.						
The form doesn't have vertices.						
The form has a tip.						
The form has 8 vertices.						

I notice the following:

_____

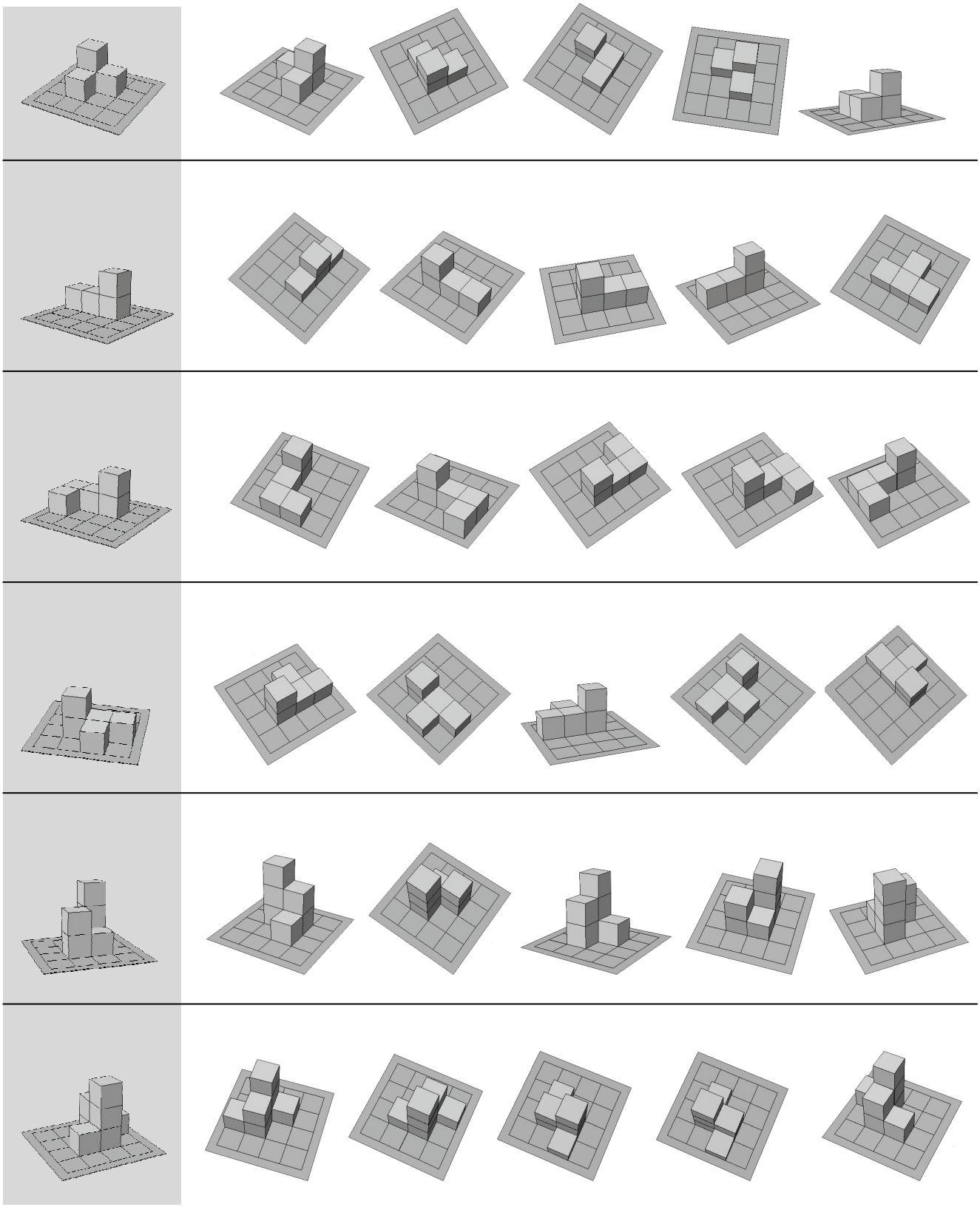
_____

Name: Date:

WS 9

Moving Solid Forms Mentally

1. Color the forms in each row that are identical to the first one.
   In each row, two of the forms don't match.

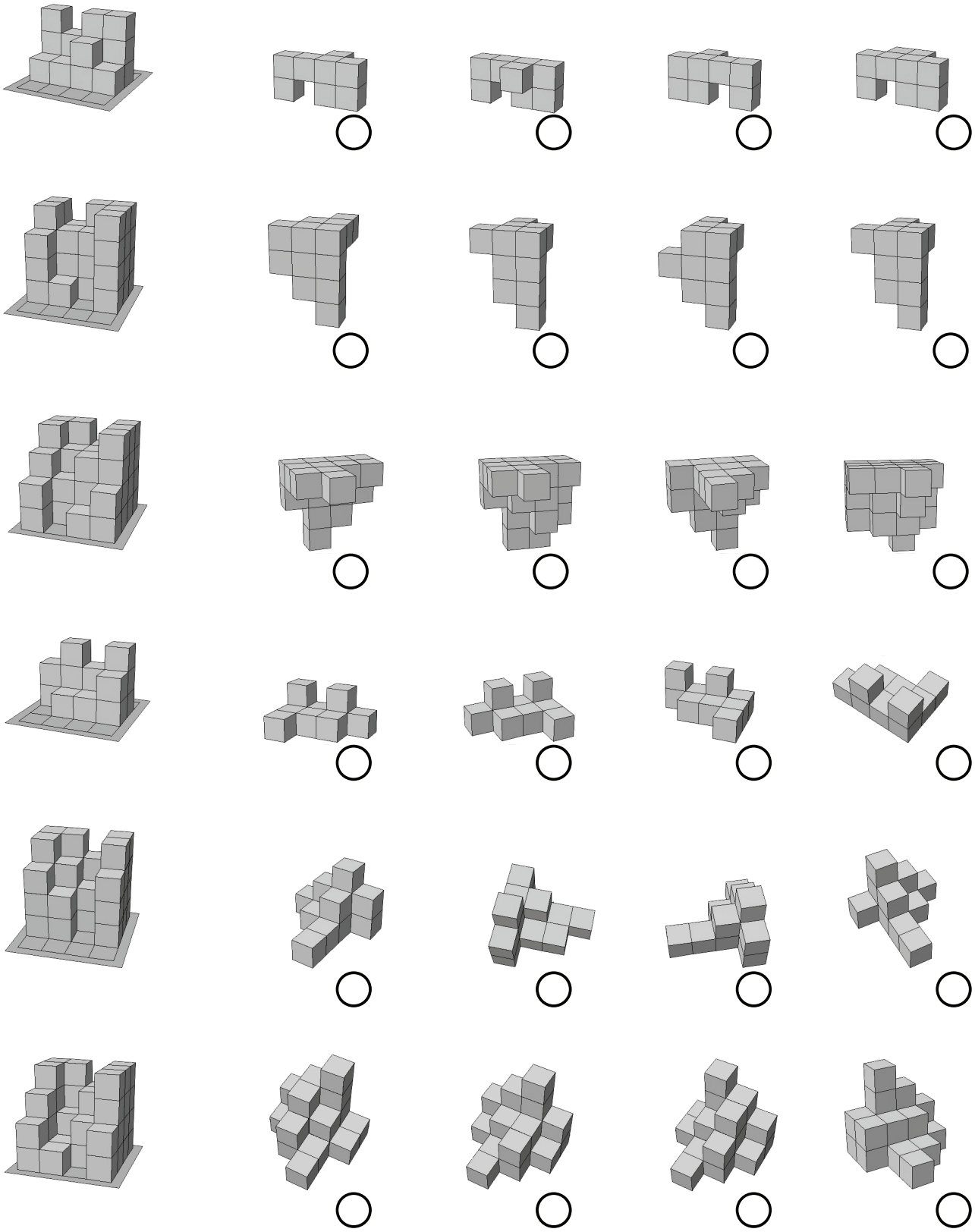


Name: Date:

WS 10

Walls

1. Which part fits the gap? Tick it.



3 Quapento

Three-dimensional Orientation – Positioning with Quadrominoes and Pentominoes

P. Ihn-Huber

Reference to Standards and Competences

Spaces and Shapes – Orienting in spaces:

- Recognizing, describing and using three-dimensional relationships

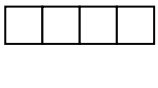
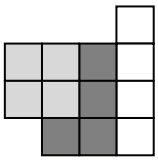
Spaces and Shapes – Recognizing, naming and depicting geometric figures:

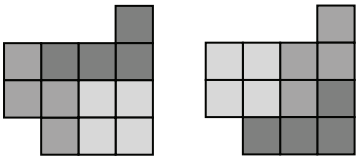
- Arranging solids and planes by characteristics and matching them with mathematical terms
- Making and analyzing models of solids and planes (by building, arranging, combining, disassembling, cutting out, folding…)

Background and Didactic Commentary

Quadrominos (plane figures made of 4 equal-sized squares) and Pentominos (plane figures made of 5 equal-sized squares) can easily be made by the children. Quadrominos and Pentominos are considered to be different when they cannot be brought into the same position neither by rotating nor flipping. There are 5 Quadrominos and 12 Pentominos (see WS 11).

These 17 forms can be put together in many different combinations. In doing so, new figures emerge that can again be presented graphically in different ways:

The figures  and  can be combined to form: 

This new figure can also be built in a different way using three Quadrominos: 

Methodological Advice

Quadrominos and Pentominos can be drawn on a squared grid or created by gluing quadratic sticky notes or small squares of paper together. The second method has the advantage that the constructed figure can be easily arranged as well as flipped and rotated. Each child can make several different figures individually at first, then all of the figures can be collected in one group for the entire class. Identical figures should then be arranged on top of each other. The teacher can give hints on how many different figures of a particular type are possible. This way all possible kinds of Quadrominos and Pentominos can be found in class.

The children should then discover new figures by putting together three Quadromino or Pentomino pieces, which can later be copied by other children. To make it easier to copy, the child should note how many Quadrominos and Pentominos he or she has used in each new figure.

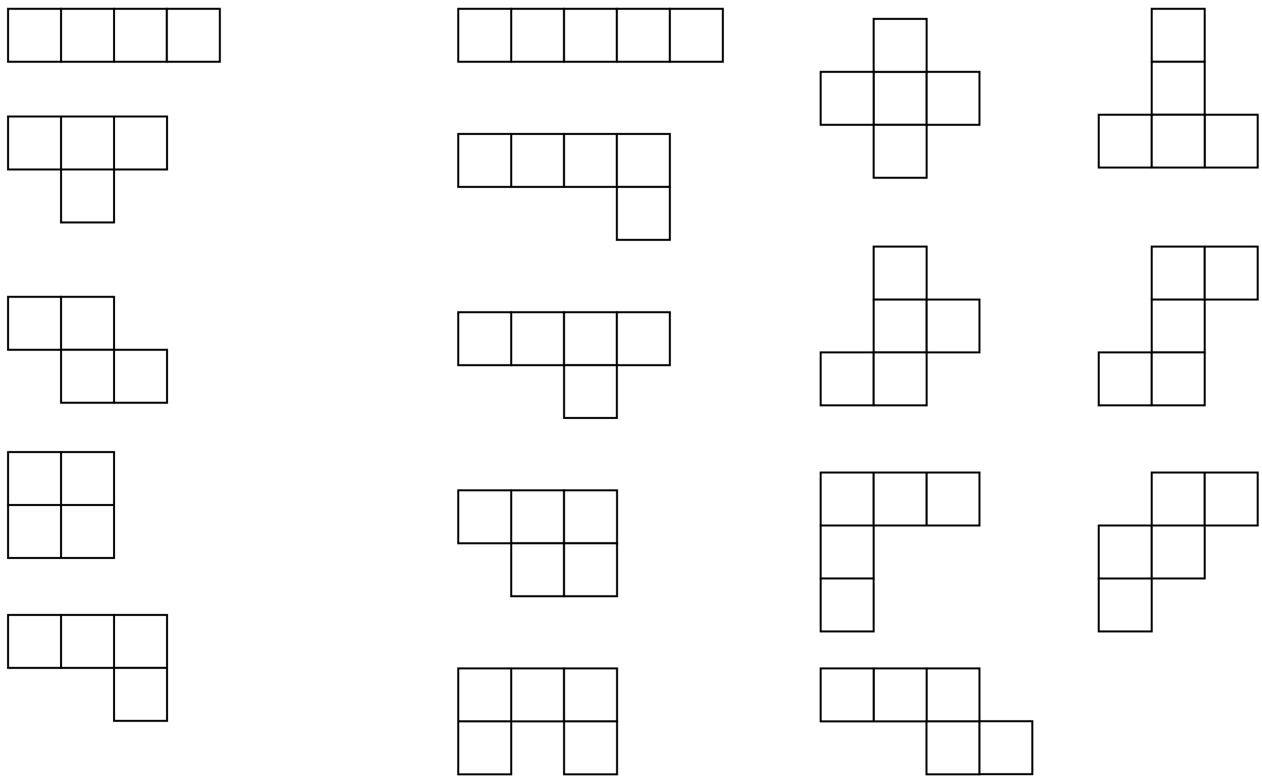
By putting together different pieces new figures emerge. Tentatively the children should solve tasks 2 and 3 on WS 13. In doing so they should find different options with each figure. For these tasks it is advised to photocopy the Quadrominos and Pentominos for all children onto stronger paper or card stock, so that the children can physically work with the Quadrominos and Pentominos (see WS 11).

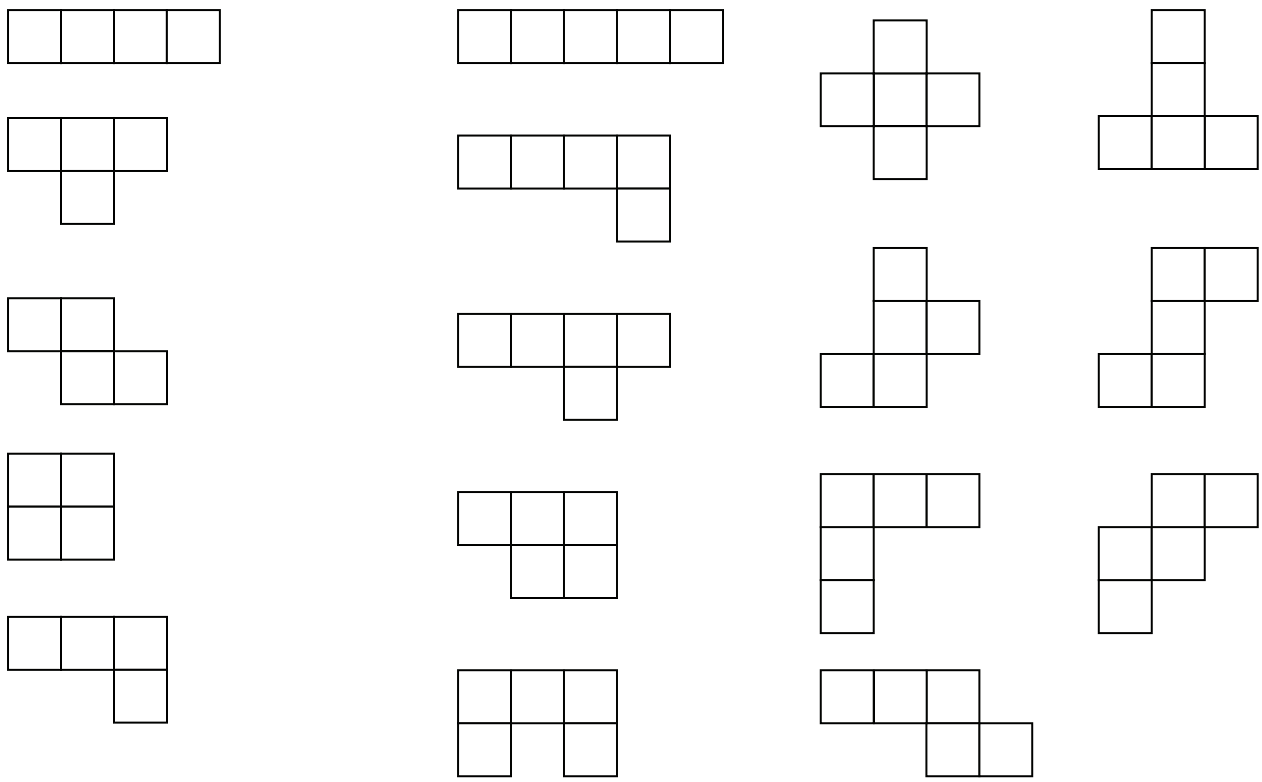
For the activity where the children make riddle cards themselves (WS 14), it is important that on the side of the riddle only the outline of the new figure is sketched, while on the solution side the used Quadrominos and Pentominos should be marked in different colors.

The last task “Lining with Given Forms” (WS 15) should be printed on individual cards. The children can make a competition out of the activity. Each child draws a card. The fastest to create the figure requested on the card wins, next game. Afterwards the children can invent new task cards themselves.

Name: Date:

Worksheets for Quadrominos and Pentominos



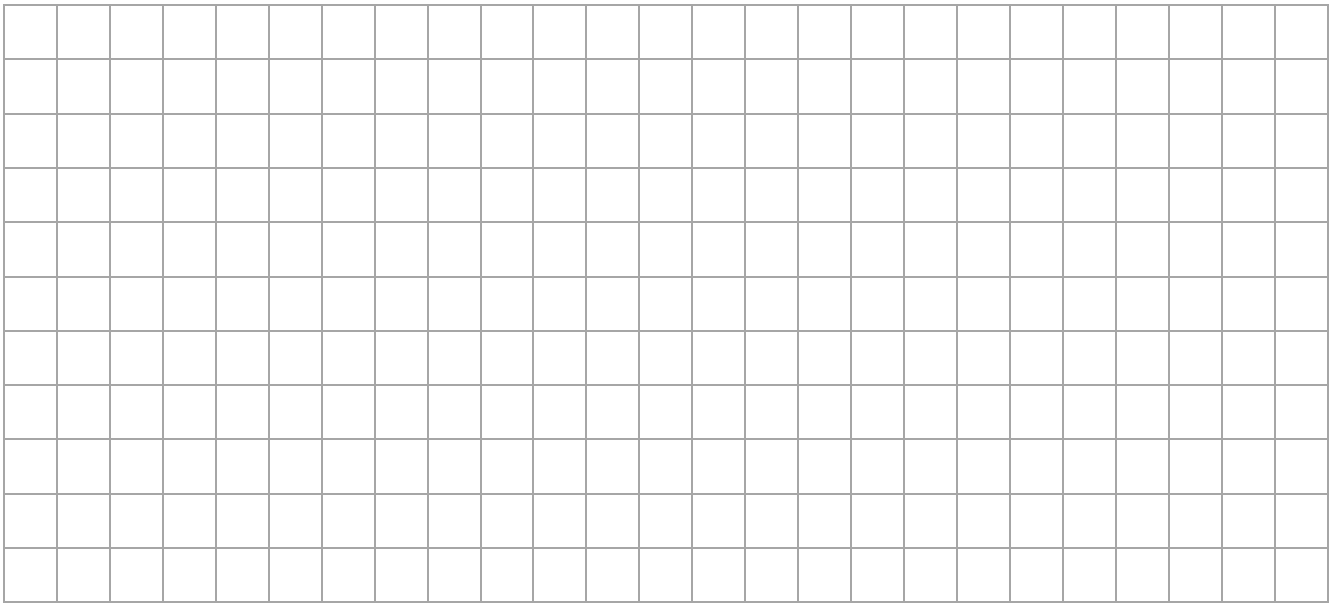


Name: Date:

WS 12

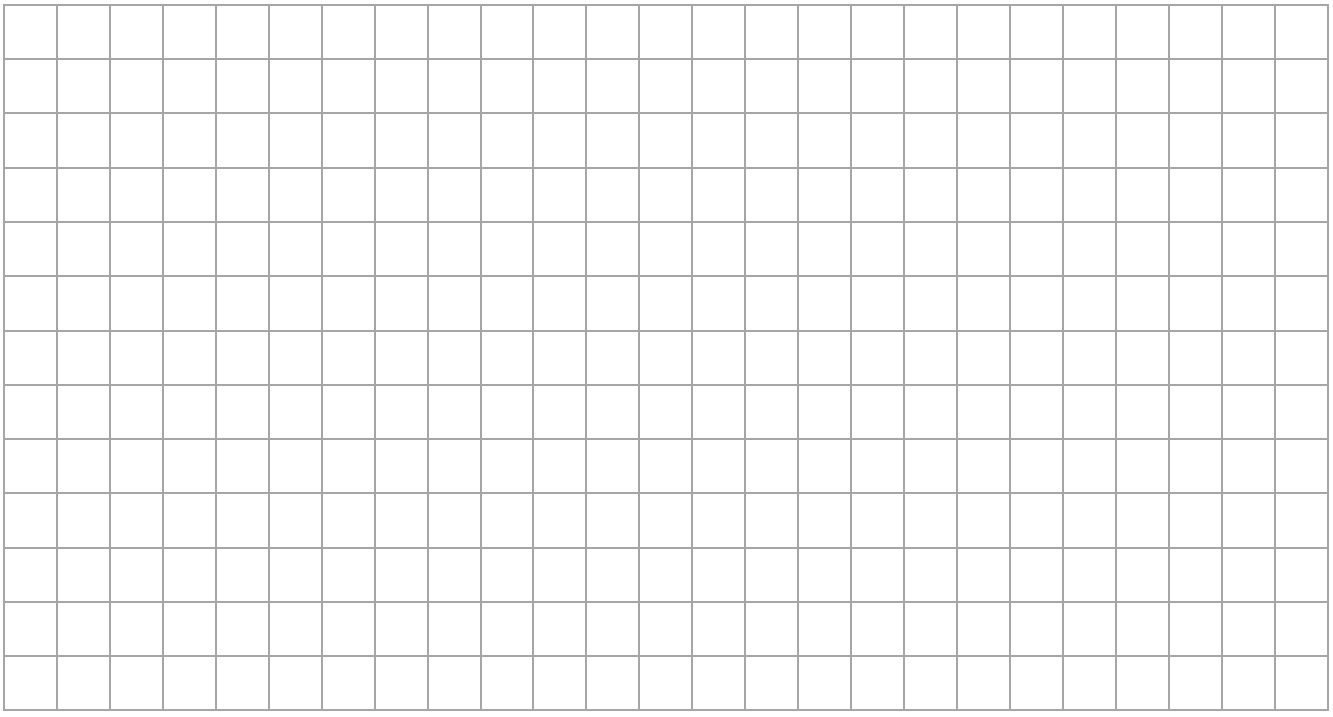
Drawing Quadrominos and Pentominos

1. Draw the Quadrominos.



2. Glue together your 3 favorite Quadrominos using square sticky notes.
   Collect all possible Quadrominos in class. How many different ones do you find?

3. Draw the Pentominos.



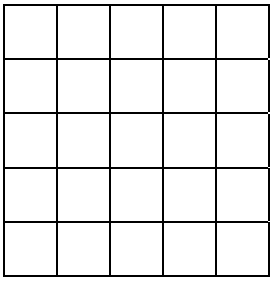
4. Glue together your 3 favorite Pentominos using square sticky notes.
   Collect all possible Pentominos in class. How many different ones do you find?

Name: Date:

WS 13

Building New Figures

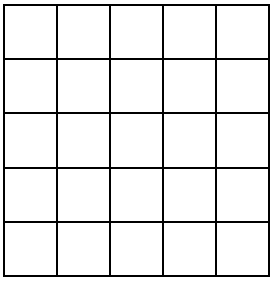
1. Choose 3 Quadrominos or Pentominos. Put them on the grid and color the squares each on covers. Write down how many Quadrominos or Pentominos you have used.



I have used:

___ Quadrominos

___ Pentominos

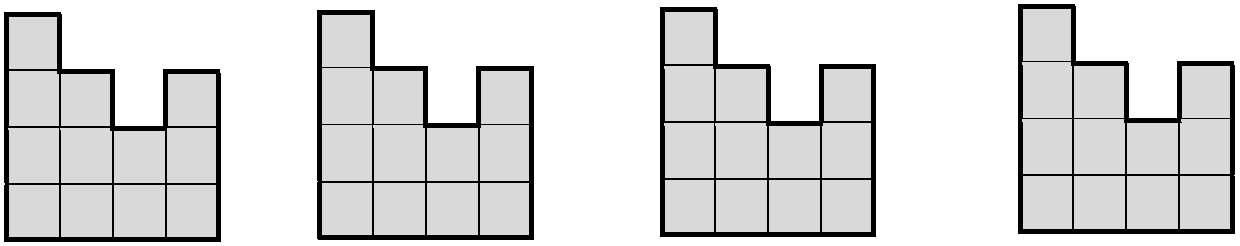


I have used:

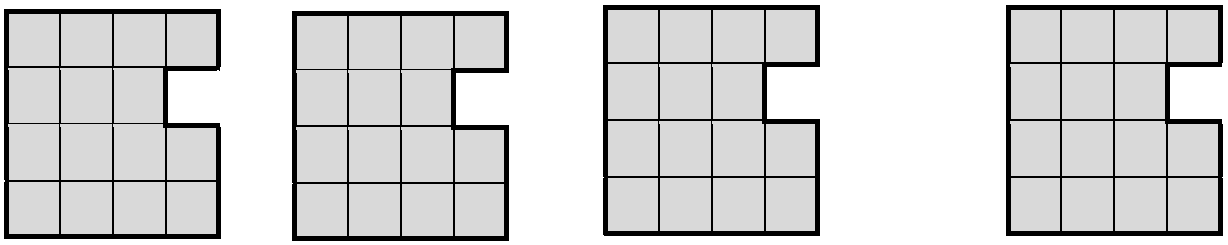
___ Quadrominos

___ Pentominos

2. Cover this space with 3 Quadrominos.
Find different possibilities.



3. Cover this space with 3 Pentominos.
Find different possibilities.



Name: Date:

WS 14

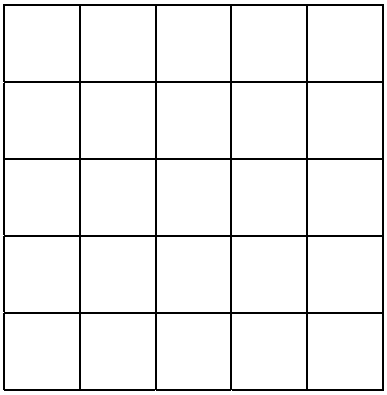
Riddle Cards

Riddle by _____

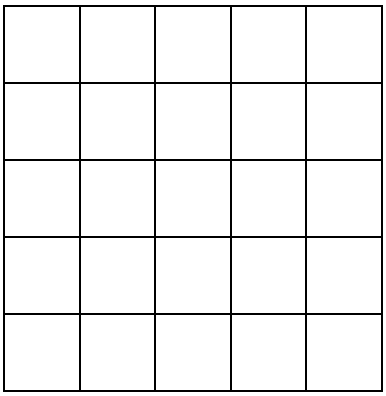
I have used:

___ Quadrominos

___ Pentominos



Solution

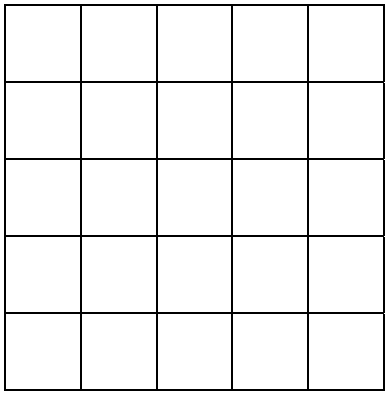


Riddle by _____

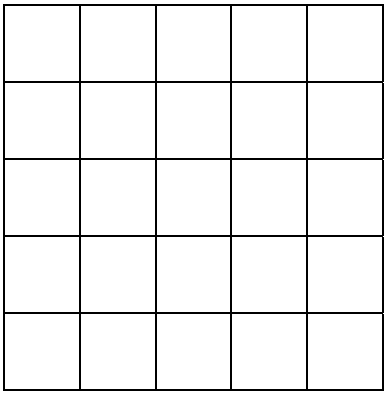
I have used:

___ Quadrominos

___ Pentominos



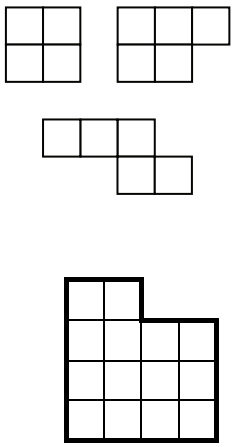
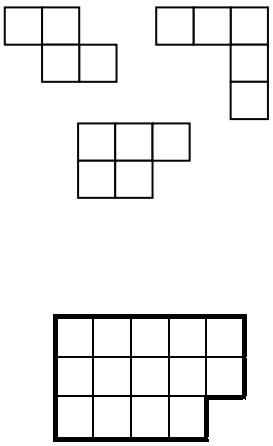
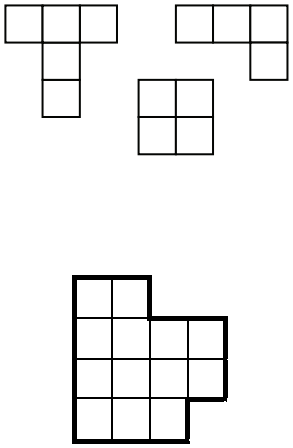
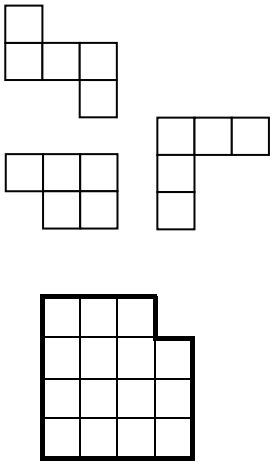
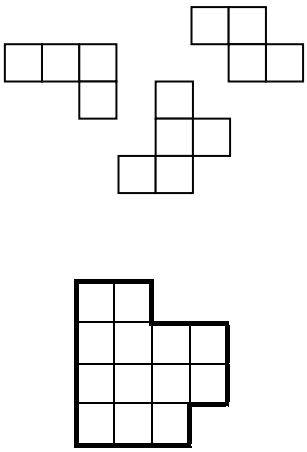
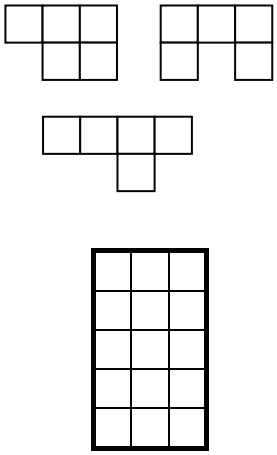
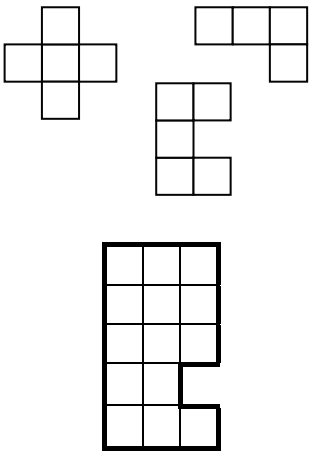
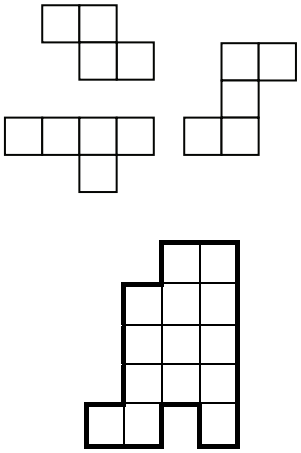
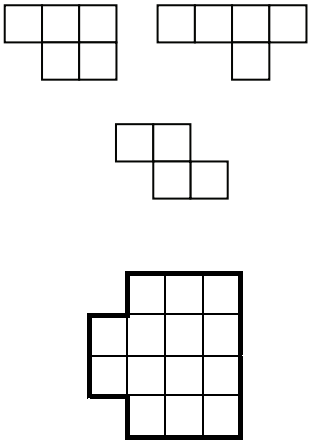
Solution



Name: Date:

WS 15

Lining with Given Forms

Use these forms: 	Use these forms: 	Use these forms: 
Use these forms: 	Use these forms: 	Use these forms: 
Use these forms: 	Use these forms: 	Use these forms: 

4 Buildings from Different Solids
Solid Forms

E. Kick

Reference to Standards and Competences

Spaces and Shapes:

- Possessing spatial awareness
- Recognizing and using spatial connections (plans/maps, perspectives)
- Relating two- and three-dimensional depictions of buildings (drawing plans for buildings)
- Arranging solid forms by characteristics

Background and Didactic Commentary

The activity of building with different three-dimensional geometric forms contributes significantly to the extension and consolidation of a pupil's spatial awareness. When starting school some children are already able to draw on experience they have gained by playing with building blocks. Thus some pupils know that building blocks can be rotated and flipped and which forms are suitable for building different shapes, such as a tall tower. We can call on that knowledge in this section.

For these activities, children like to bring their building blocks to school. Activities like arranging, sorting out, describing and finally naming important solid forms are added to the building activity. Asking pupils to describe a building made from a limited number of geometrical forms orally or in writing so that the building can be recreated by a classmate is a real linguistic challenge. The ground plan where the numbers of the necessary cubes, cuboids, ... are filled in may be a result. Another activity asks pupils to visualize a building from all sides and relate this to the three-dimensional figure. In doing so, the pupils succeed in transferring a three-dimensional building into a two-dimensional perspective.

Methodological Advice

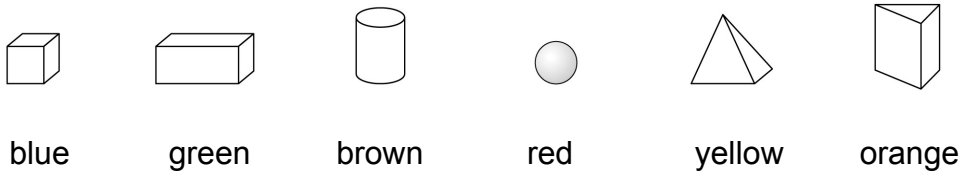
Everyday objects with basic geometric forms, building blocks and packaging are suitable materials to help pupils make a connection with everyday reality and allow the children to work using concrete actions. The pupils should describe and sort the objects they have brought to class, using more and more mathematical terminology (edge, vertex, face, prism, ...). In WS 16 the pupils are expected to recognize geometric forms and characteristics and show this by coloring them in with the same color. The goal of WS 17 is to recognize geometric forms despite their altered position in a space. Afterwards it is recommended to let the children build and possibly color their own dream buildings for the village "Geotown" using the various kinds of packaging materials. Riddle cards, written by the children, can be used to find a specific building in the village "Geotown". WS 18 shows how this subject matter can be linked with combinatorial questions. WS 19 deals with buildings and their ground plans. While in task 1 the children have to match the ground plans to the according buildings, in task 2 they have to draw their own ground plans. Before working on WS 20, the pupils can build small buildings together (group work) and view them from all sides. In so doing the children try to describe exactly what they recognize from each perspective and record this in a picture. Past experience shows it is best not to use more than four bricks for this exercise.

Name: Date:

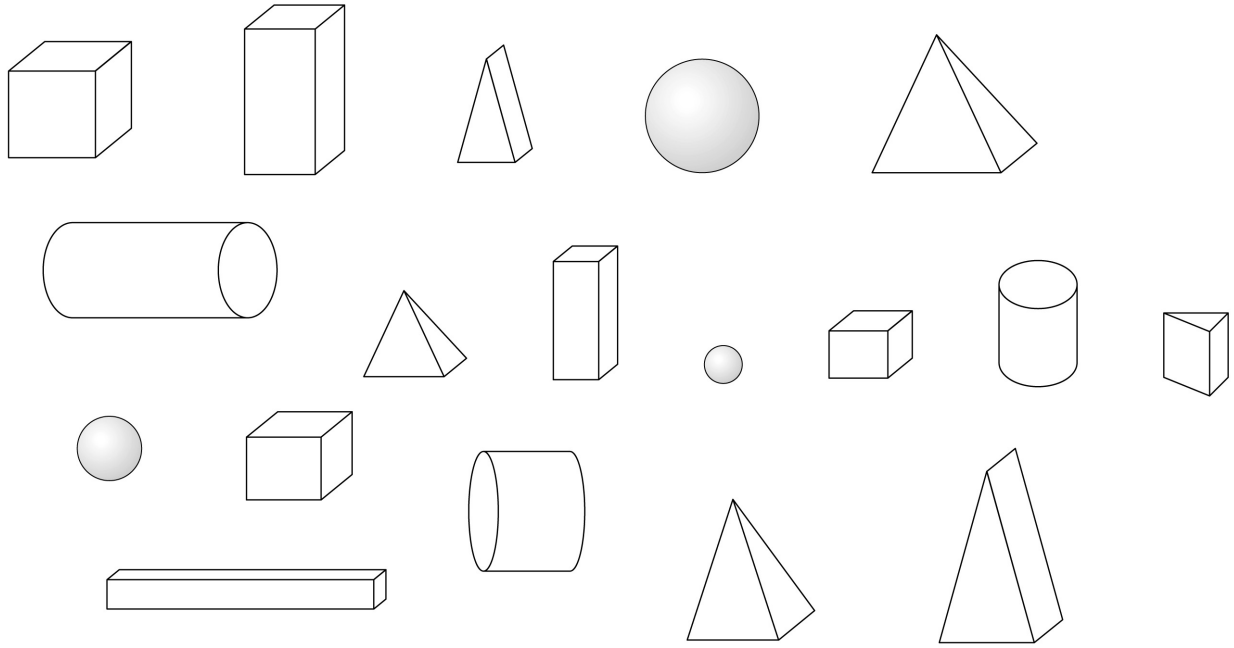
WS 16

Building Sites in Geotown

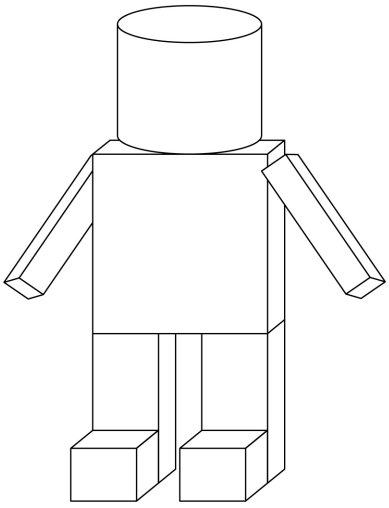
1. Paint in color.

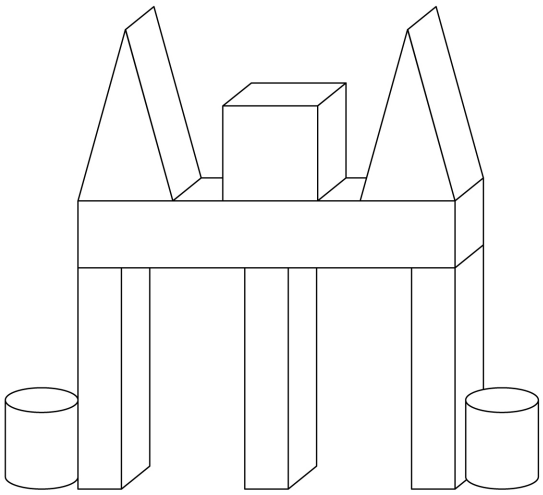


blue green brown red yellow orange



2. Color the forms in the figure and the building.



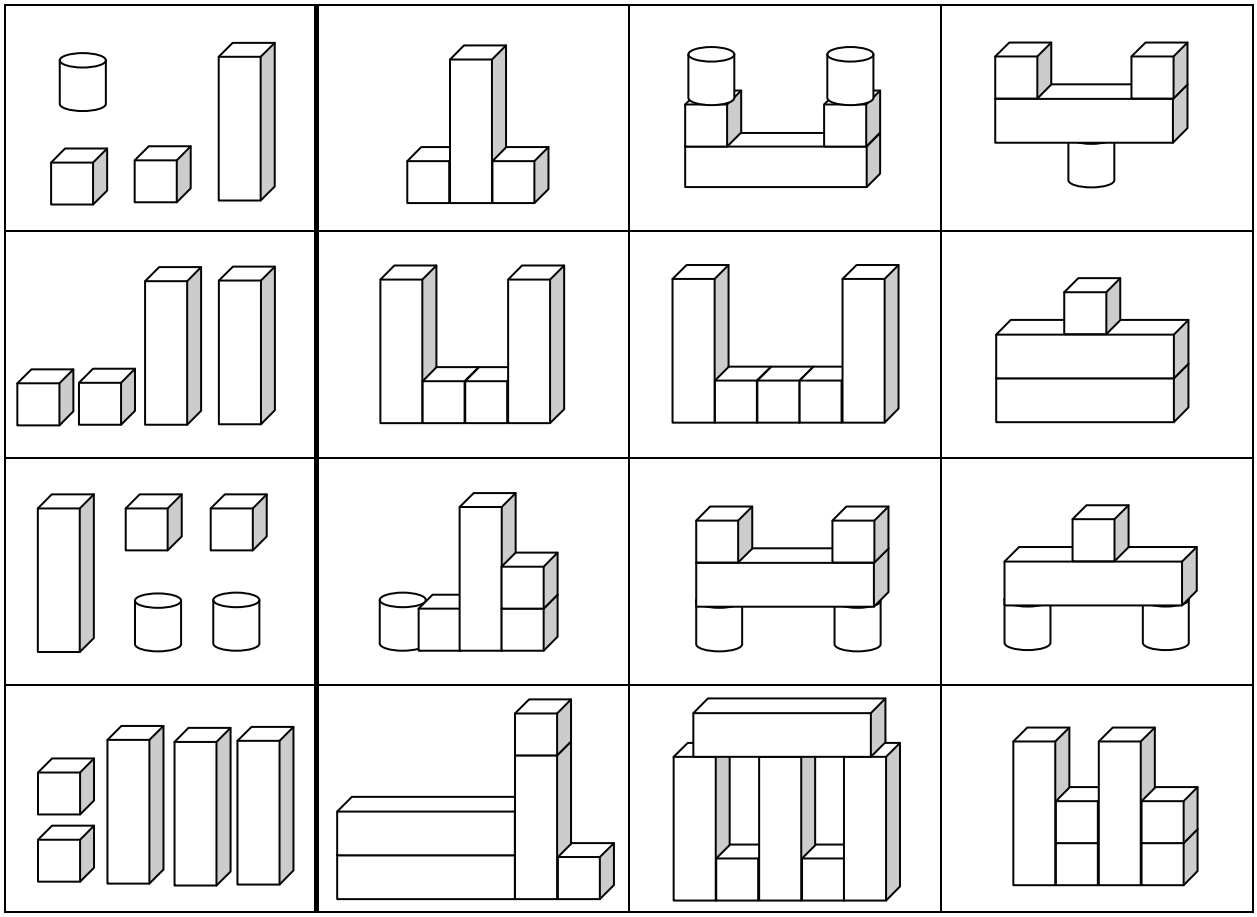


Name: Date:

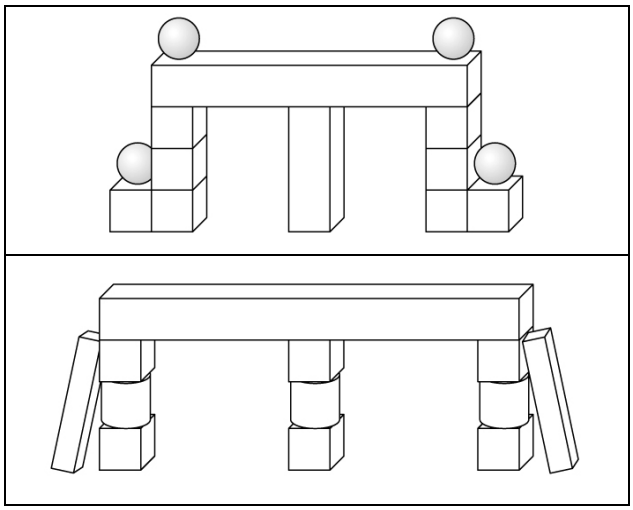
WS 17

Building with Geometric Forms

1. In the first picture you find the forms you can use.
   Which buildings can you make from these forms? Color them.



2. Gates have to be built at the entrance of town. Which geometric forms do you recognize? Write down how many times each basic form has been used.

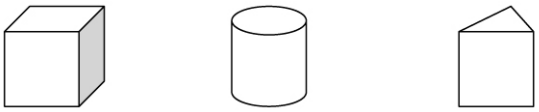


Name: Date:

WS 18

Combinatorial Questions

1. Bricklayer Geolux would like to use 3 different solid forms for each of his gateposts: a cube, a cylinder and a prism.



How many different gateposts can he build from them? Sketch them.

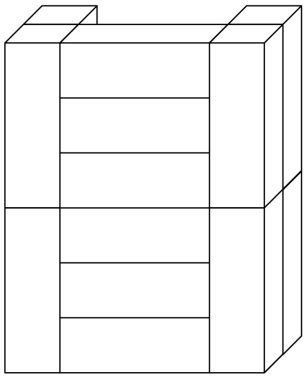
2. Bricklayer Geolux thinks:
„I'll replace the cylinder by a cube. So now I have two cubes and a prism."
Does bricklayer Geolux now have more or less possibilities?
What do you think? Check.

Name: Date:

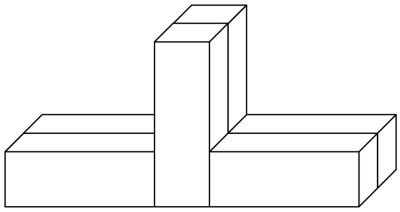
WS 19

Buildings and their Ground Plans

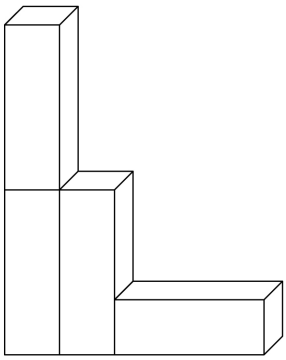
1. Match the buildings with the ground plans.



1	1	1
1	1	1

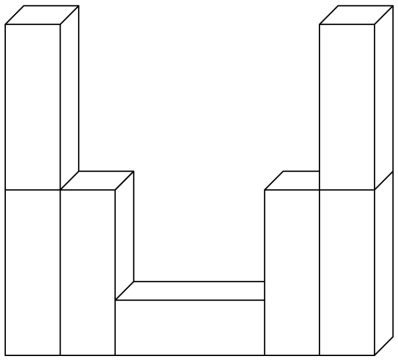


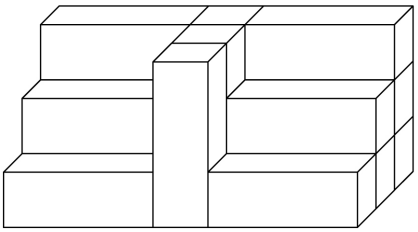
2	1	1

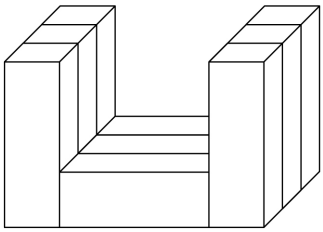


2		2
2	6	2

2. Sketch the ground plans for the buildings.



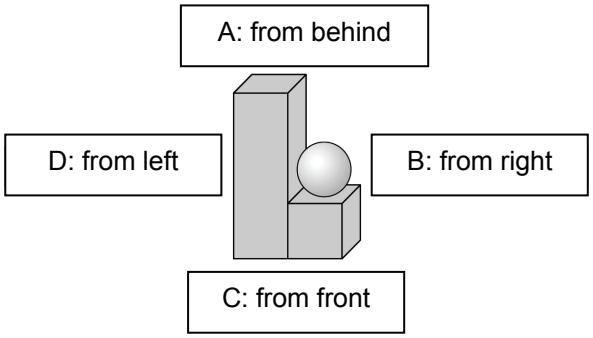


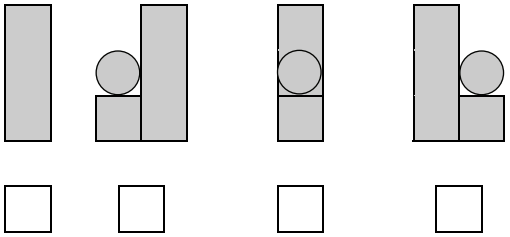


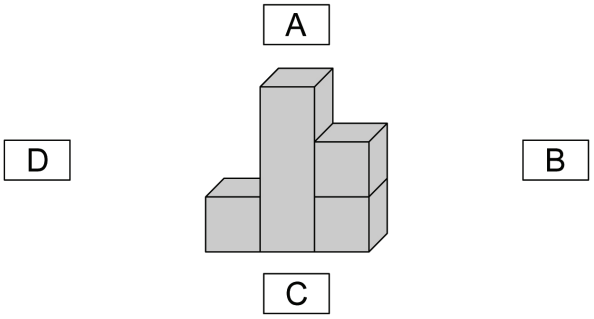
Name: Date:

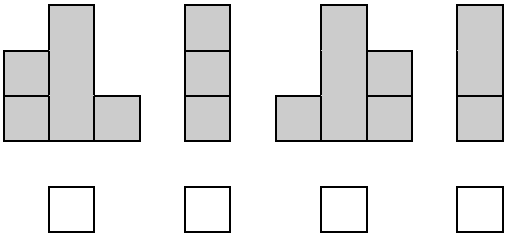
Views from All Sides

1. The buildings are photographed from all sides.
   From which point-of-view were the photos made?
   Write down the correct letter in the box.

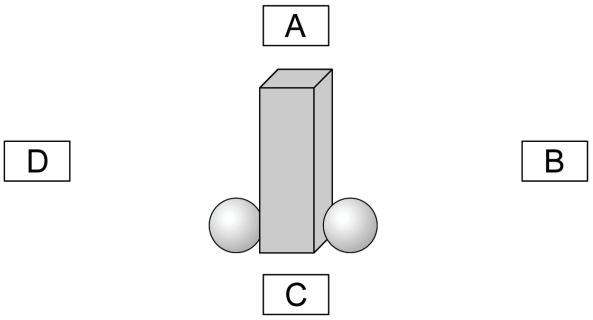


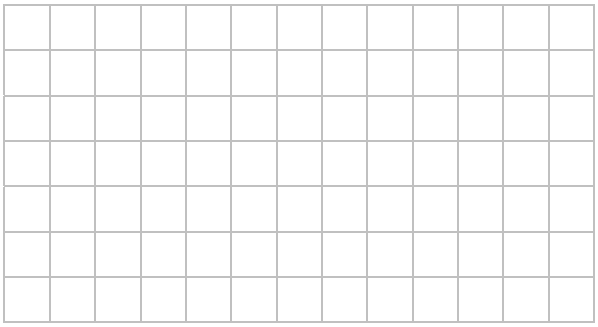




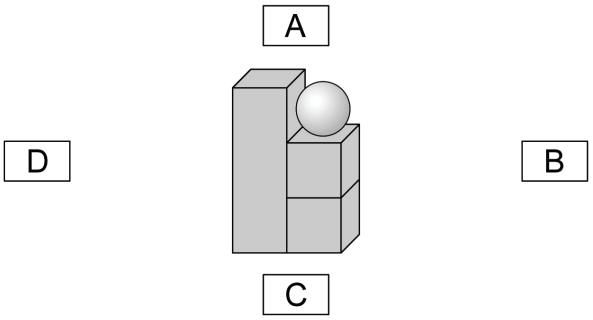


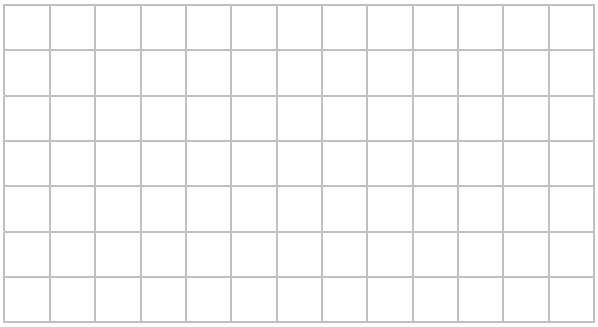
2. Now you make the photographs! Draw your building from all sides.





What do you notice?





5 Three-Dimensional Drawings
Solid Forms

H. Gutmann

Reference to Standards and Competences

Orienting in Space:

- Possessing spatial awareness
- Recognizing, describing and using three-dimensional relationship
- two- and three-dimensional graphics of buildings (e.g. putting cubical buildings into relation)

Recognizing, sketching and naming geometric figures

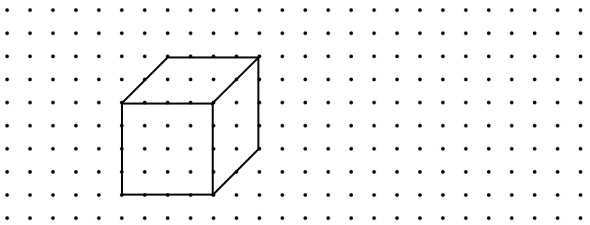
- Drawing sketches with aids as well as freehand drawings

Background and Didactic Commentary

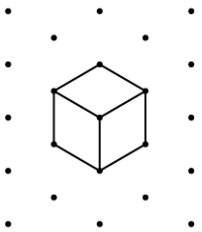
Descriptive geometry deals with mapping three-dimensional spaces onto a plane, specifically the graphic plane. To do so, the mapping procedure has to be done in such a way that the three-dimensional facts can be displayed on the drawing plane adequately. The pupils need to be able to solve tasks regarding the drawing and draw correct conclusions regarding a three-dimensional form from the sketch. Depicting a three-dimensional form is difficult, not only for children. But still, children in primary schools can be acquainted with this skill step by step. Often they already have experience with their own drawing attempts, crafting instructions or templates for copying objects with geometric forms.

Sketching oblique views is the most challenging task in geometrical drawing. To display forms three-dimensionally there are basically two ways of projecting: the central projection (copies the natural visual process, in which edges of the solid move towards a vanishing point) and the parallel projection (in which parallel edges of the solid can be drawn in a parallel way). In primary school only one special case of the parallel projection should be addressed – the so-called *cavalier projection* – as a way of displaying three-dimensional drawings. With cavalier projection there is a strong relationship between the three-dimensional object and its graphic display. Parallel straight lines are also displayed in a parallel way in the drawing, and most of the length proportions remain the same. While the lengths of the sides for width and height are shown in proportion 1:1, the lines moving towards the back are shortened.

The drawing of oblique views can be done in a square grid (pict. 1) as well as in a triangular grid (pict. 2). In the latter, all sides of the cubes are drawn in the same length (1:1:1), but the quadratic faces of a cube appear as parallelograms.



Pict. 1: Cavalier projection in a square grid



Pict. 2: Cavalier projection in a triangular grid

Methodological Advice

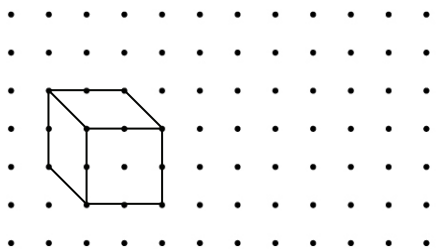
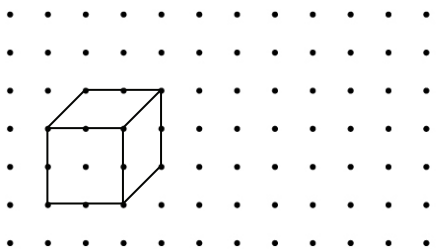
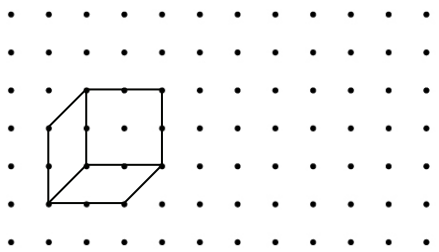
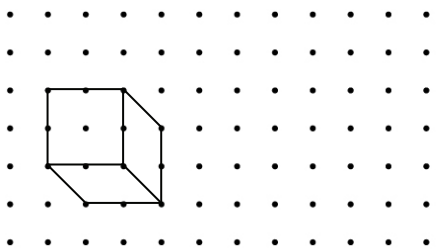
Useful preparatory activities for this unit would be geometric exercises and free building with cubes. Wooden cubes or cubes that can be plugged together are both suitable.

Name: Date:

Building and Drawing 1

1. Put a wooden cube on your table.
   Sit in front of it so that your eyes are on the same height as the cube.
   How many faces do you see from this view? _____
   Now change your position. How many faces do you see now? _____
   Next touch all the edges that you can see without moving yourself.

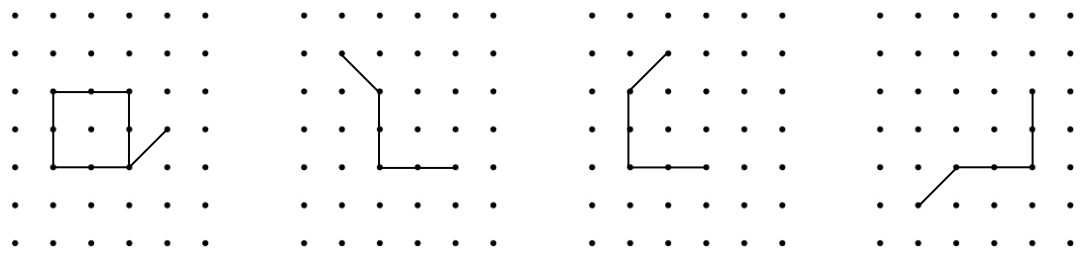
2. A dotted grid can be helpful for drawing using an oblique view.
   Copy the given solid form in the grid.

   Not all the drawn lines are of the same length. What is a possible reason?

   _____

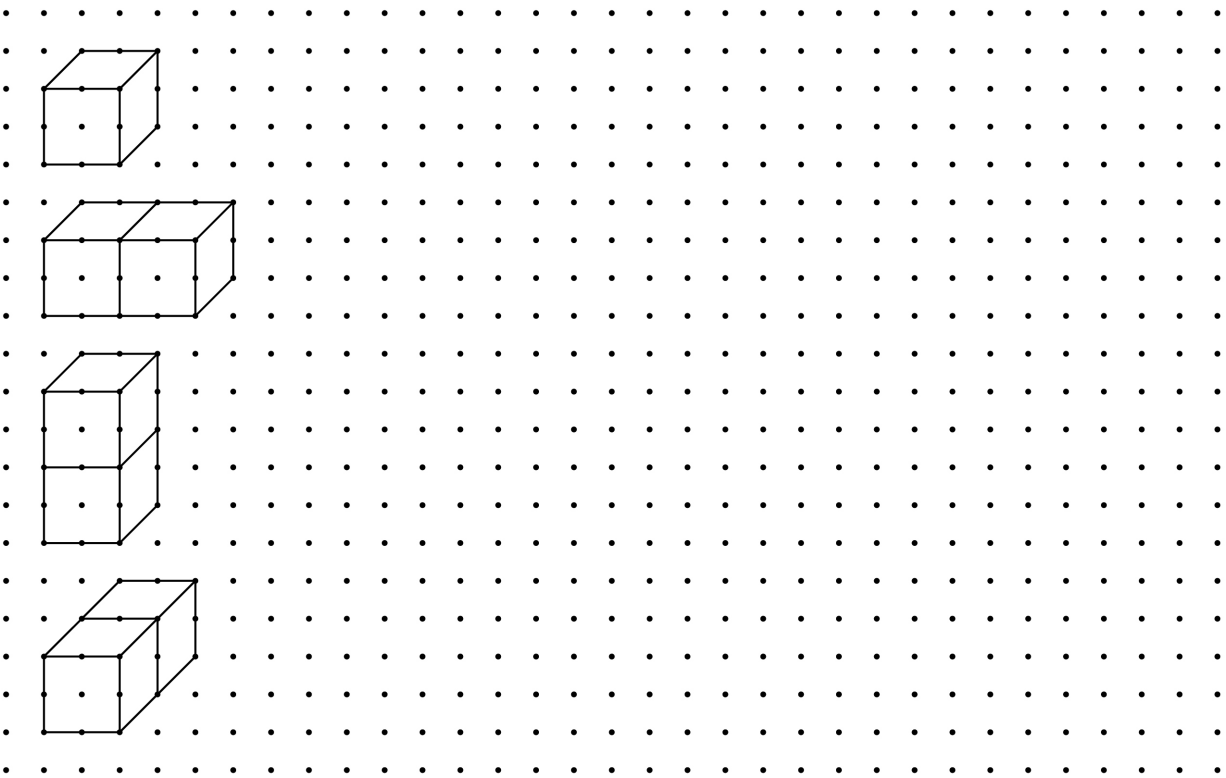
3. Complete the missing lines. A cube should emerge.



Name: Date:

Building and Drawing 2

1. You can also make columns from several cubes. Draw using an oblique view:
   - one cube
   - two cubes next to each other
   - two cubes on top of each other
   - two cubes behind each other



2. Think up more varieties and draw them using an oblique view. You can build your ideas with cubes first.

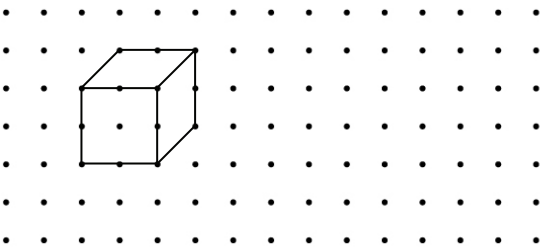
Name: Date:

WS 23

Modifying an Oblique View

1. Make your own building from 4 cubes. Sketch it.

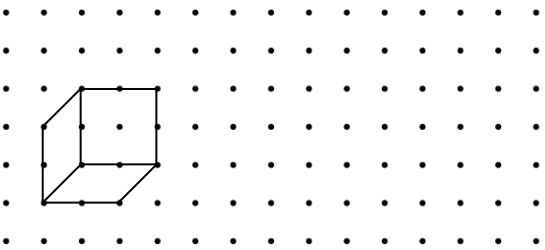
2. Change the given cube. Draw the new form as instructed. Describe.



Make the front face wider by 2 squares. What do you notice?

_____

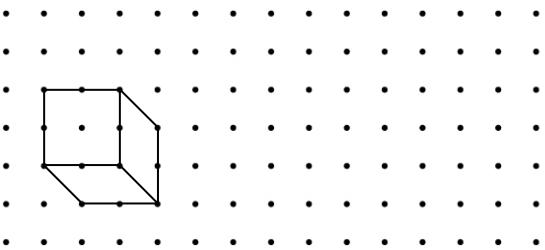
_____



Enlarge the front face of the cube by 1 to the right and 1 upwards. What do you notice?

_____

_____



Minimize the front face of the cube by one square in width and at the same time enlarge it by one square upwards. What do you notice?

_____

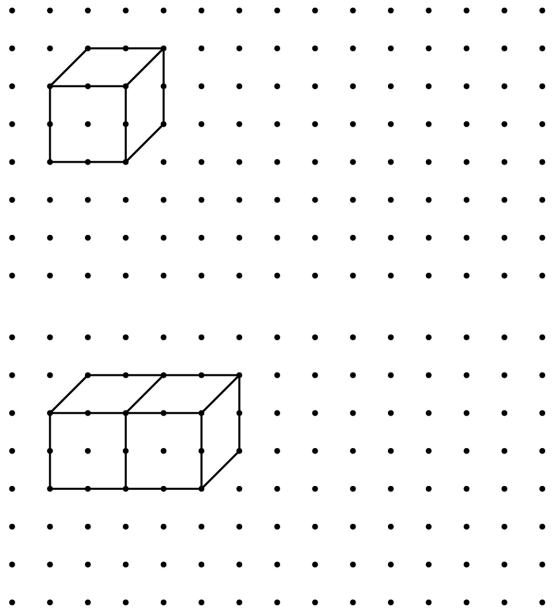
_____

Name: Date:

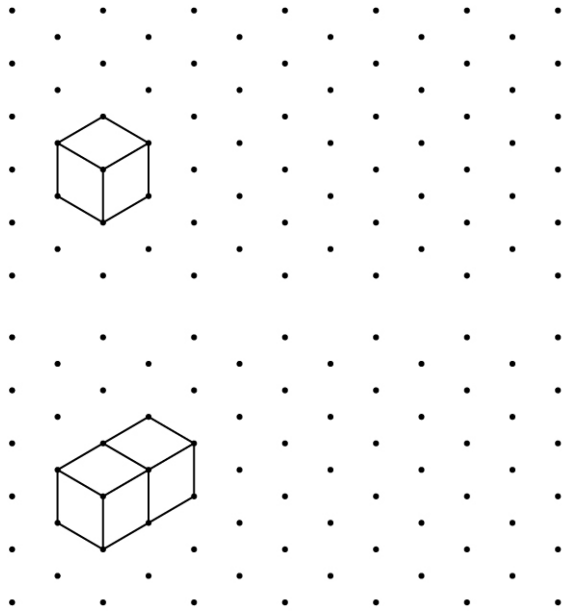
Drawing with Different Grids

1. Draw the cubes into different kinds of grids.

square grid



triangular grid

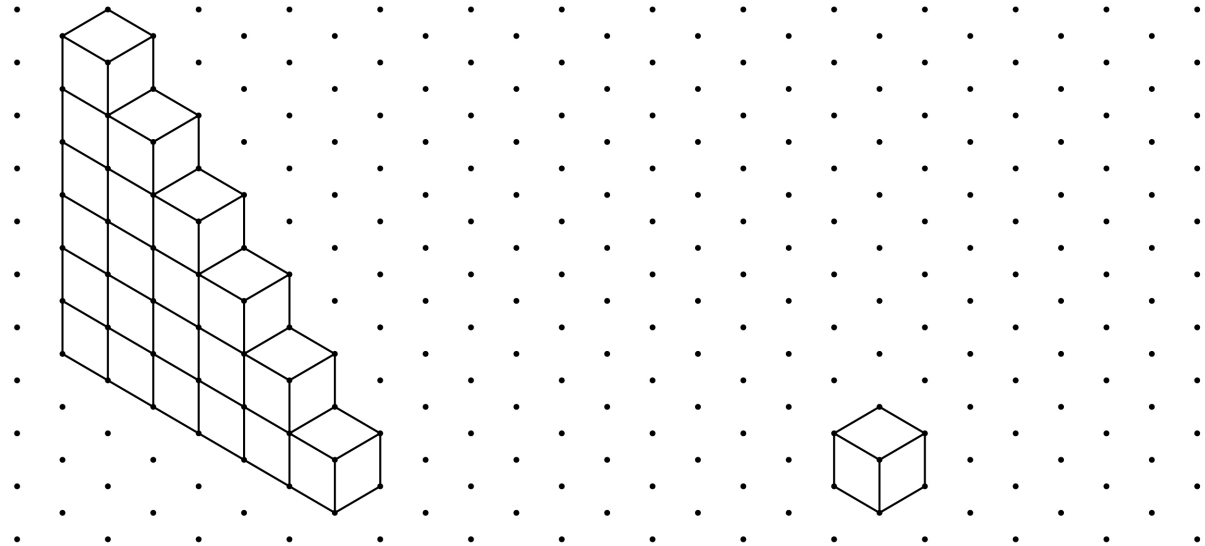


What do you notice?

_____

_____

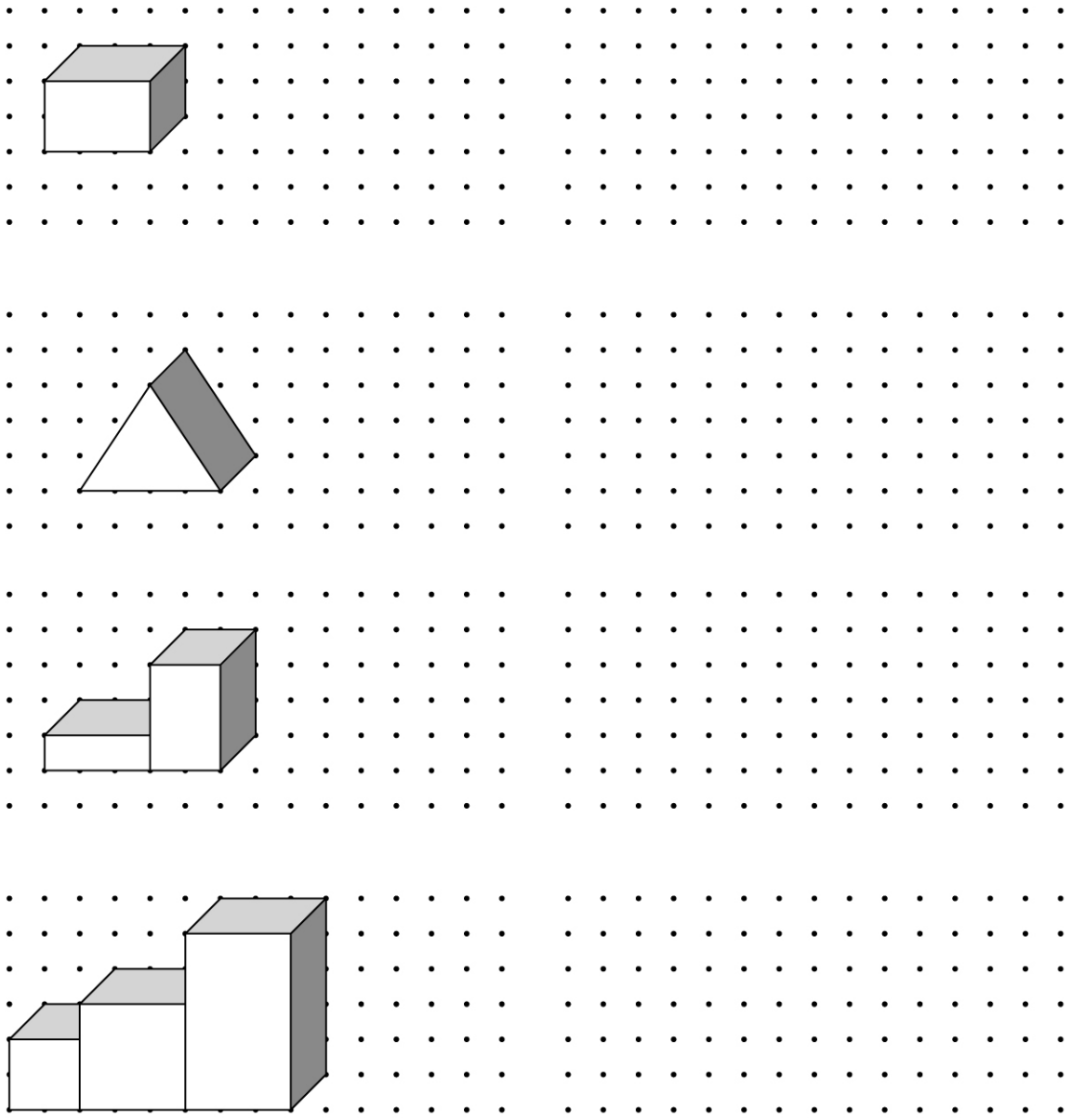
2. Now try to draw the stairs made of cubes into the triangular grid.



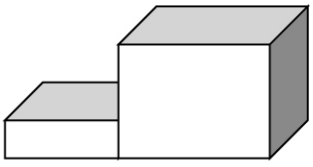
Name: Date:

Drawing With and Without Grids

1. Draw the following solid into the grids.



2. Now draw the form without a grid.



6 Quadruples
Solid Forms – Building with Cubes

P. Ihn-Huber

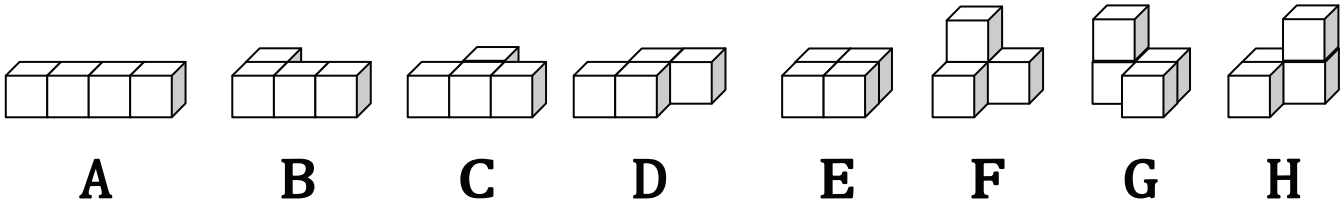
Reference to Standards and Competences

Spaces and Shapes – Orienting in Space:

- Possessing spacial awareness
- Making three-dimensional connections
- Arranging solids and plane figures by characteristics and matching them with mathematical terms
- Creating and analyzing models of solids and plane figures

Background and Didactic Commentary

Assembling cubes to form new shapes, classifying these shapes and constructing plans for such shapes develops spatial awareness and geometrical skills.
In the following tasks, the playful aspect is just as important as the later recognition of a classification, completeness or representation of buildings made from cubes.
There are eight different ways to connect four cubes together:



All other possibilities can be found by rotating these eight combinations.
Due to the small number of possible combinations, it is reasonable to let the children find all possible combinations by themselves, to compare quadruples, to recognize the same or similar quadruples and to find an argument for what is a complete list of all combinations.

Methodological Advice

The activities with the cube quadruples can be done in two to three lessons. It is important to start with a simple task where each child constructs quadruples (WS 26). By actively handling the cubes (e.g. wooden cubes), each child can experience rotating their combinations; that is to say that a quadruple may look different in various positions, but it still has the same arrangement of the cubes. Standard wooden cubes are appealing due to the material, and they are geometrically clear due to their flat faces. But the problem is that the quadruples tend to fall apart while being rotated. This can be corrected by using an adhesive, which can be used to glue the cubes together and can be later removed. A more stable option but less exact in form is to use cubes that can be plugged together.
Argumentation is of great importance when the children decide if the quadruples are equal or different while they discuss the handling of their combinations: "When I rotate my quadruple it looks exactly like yours. Therefore they are not different but the same".
If the teacher demands a set of eight different quadruples, the children have a clear goal, but perhaps frustration if they do not achieve all eight quadruples. It is more interesting to let children explore by themselves, maybe leaving the task in class for a longer time so that they can think about it further. If children are content with their results too quickly, the teacher can keep them thinking by showing a "new" quadruple.
When drafting the construction plans (WS 27 and 28), be sure to discuss in class that each quadruple has a base and that a height has to be written on each square of the base. Construction plans can be illustrated like this:

For Quadruple A:

1	1	1	1

or

4

For Quadruple B:

2	1	1

or

3	1

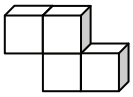
	1	
1	1	1

The total sum of the numbers in the construction plans is always four, as this sum is the number of the cubes.

Is there only one possible construction plan for a quadruple or are there more?

- There is one construction plan for Quadruples D, F, G and H (excluding reflections and rotations).
- There are two constructions plans for Quadruples A, C and E.
- There are three construction plans for Quadruple B.

Note that Quadruples B, C, D, F, G and H could also be positioned such that the base is smaller than the bird's eye perspective: e.g.



To draw up the construction plans the class must make an agreement if "floating" cubes are allowed. Does every cube have to lie on the ground or on another cube? Here children may find new construction plans for such situations, e.g. for the above quadruple:

2-1	2	1

Here "2-1" means: "There are two cubes, but the bottom one (on the ground level) is missing."
Note that (2-1)+2+1 again equals four, which is again an appropriate arithmetic description.

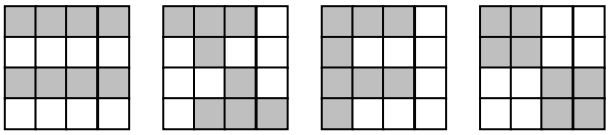
While drawing up construction plans and when bringing the class together to share their plans, the children can be encouraged to organize the plans. One possible sorting is:

- Construction plans with the highest height of one. (Construction plans for A, B, C, D and E are suitable.)
- Construction plans with the highest height of two. (Construction plans for B, C, F, G und H are suitable.)
- Construction plans with the highest height of three. (Construction plan for B is suitable.)
- Construction plans with the highest height of four. (Construction plan for A is suitable.)

Children also distinguish construction plans with 4/3/2/1 squares in the base, etc.

By doing this activity, children also understand why there cannot be more than eight distinct quadruples.
If the four cubes are arranged in a plane, the only possibilities are all four in a long row (A), one cube next to the remaining three (B and C), or two cubes next to the other two (D and E).
For combinations in which three cubes are on one level and the fourth on the level above, the fourth cube can sit on any of the three other cubes. Thus there are three possibilities (F, G and H).
As soon as the quadruple is three cubes high, the quadruple can be rotated such that it is only one cube in height.

Lining quadratic planes with quadruples (WS 29) is done by trial and error. Here the children will discover quickly that the quadruples F, G and H are not suitable, as they cover two levels in each position. With the quadruples A, B, C and E, the 4x4 squares can be lined. Quadruple D always creates squares in the corners which can no longer be covered.



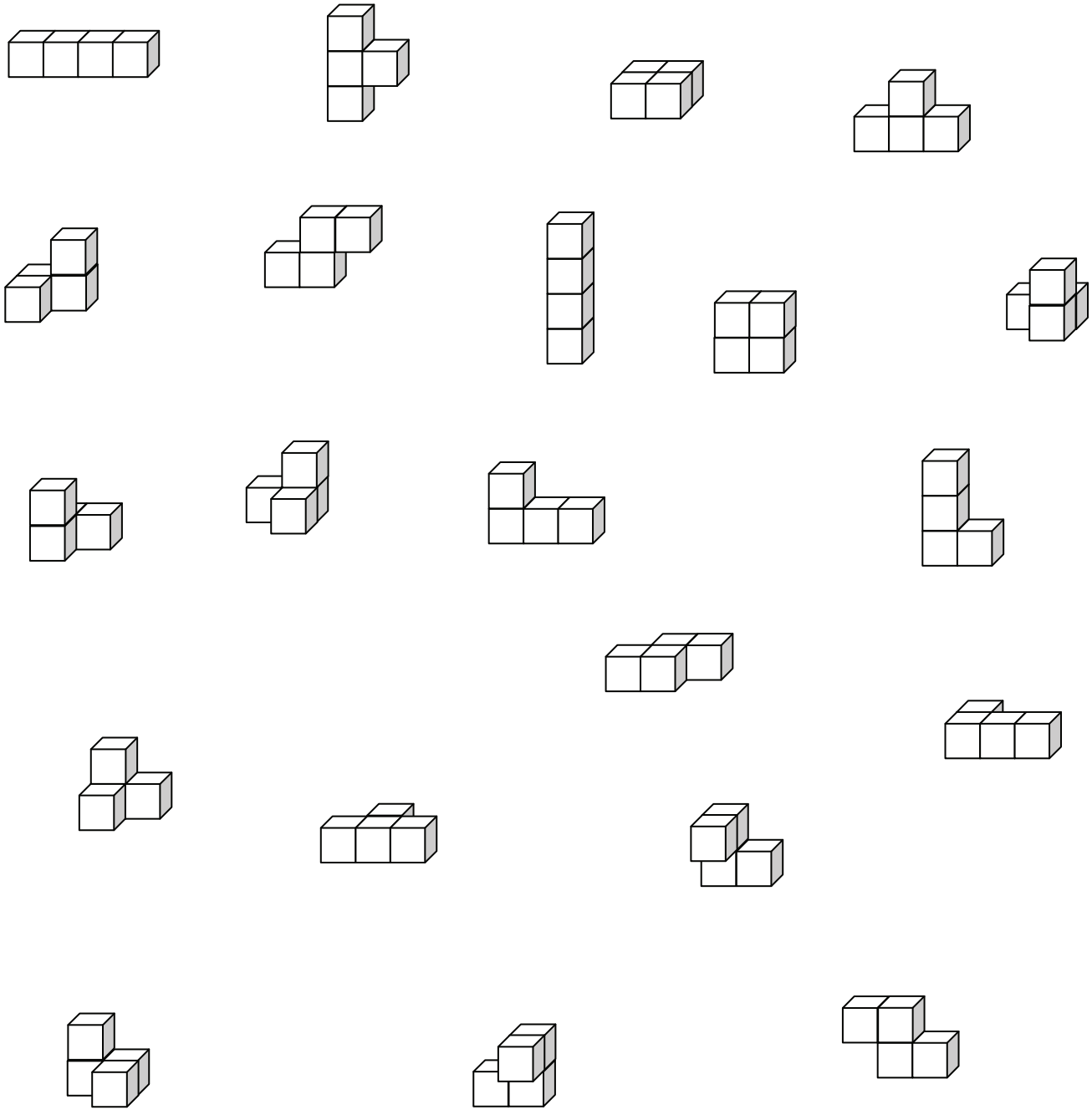
Three-dimensional drawing is practiced with the quadruples. It may be helpful if the children have the quadruple that they are drawing in front of them. It can also be helpful to color in the front faces of the cubes in one color, the right faces and other sides in different colors.

Name: Date:

WS 26

Building Quadruples

1. Make quadruples by yourself. Compare with your partner.
2. How many different quadruples did you find?
3. How many different quadruples did you find in the class?
4. Think: Which of the quadruples below are equal? Tint them in the same color.

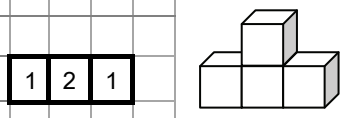


Name: Date:

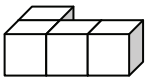
WS 27

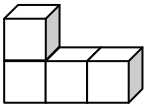
Drawing Construction Plans

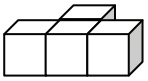
1. This construction plan is for the given quadruple.
   Draw construction plans for other quadruples.



2. Is there only one possible construction plan for each quadruple or are there more?







3. What do all construction plans for quadruples have in common?

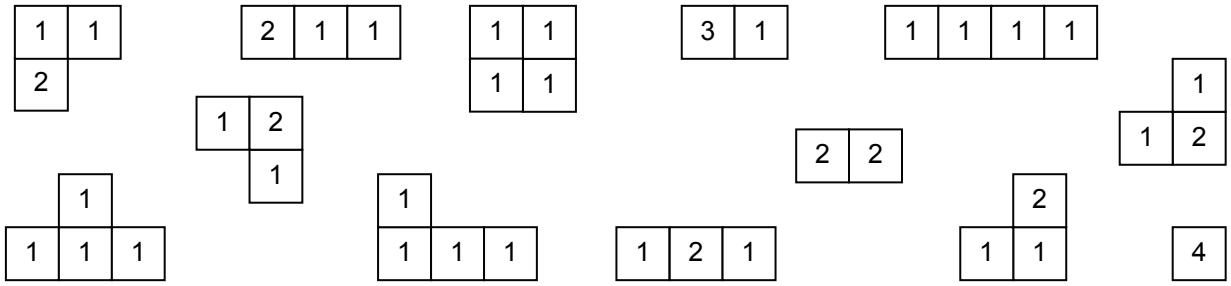
4. Try to arrange the construction plans in groups.

Name: Date:

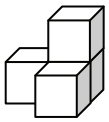
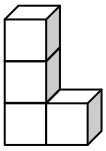
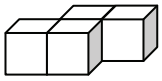
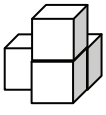
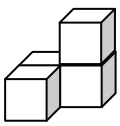
WS 28

Different Construction Plans

1. Which construction plans belong to the same quadruple? Connect the plans with lines.



2. Complete the construction plans for the given quadruples.

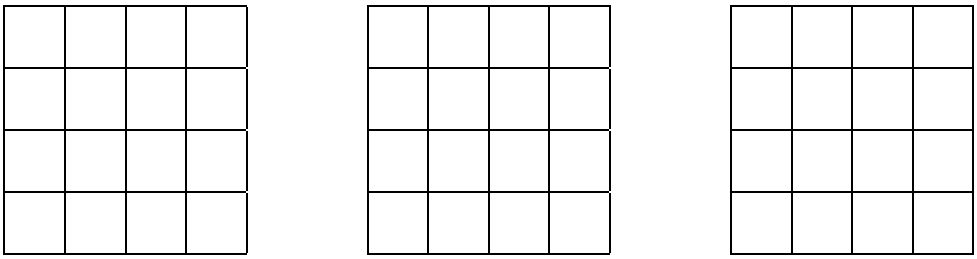
	1 2
	
	1 1
	1
	

Name: Date:

WS 29

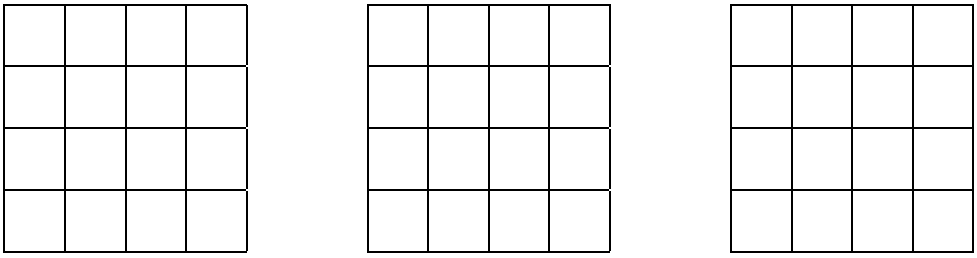
Lining Squares

1. Choose 4 different quadruples.
   Can you line out the area of a 4x4-square with them?
   Draw them in the squares below.



2. Which quadruples are suitable for lining the squares? Which are not?

3. Can you line the 4x4-squares with 4 equal quadruples?
   Sketch them.



4. Which quadruples cannot line the squares?

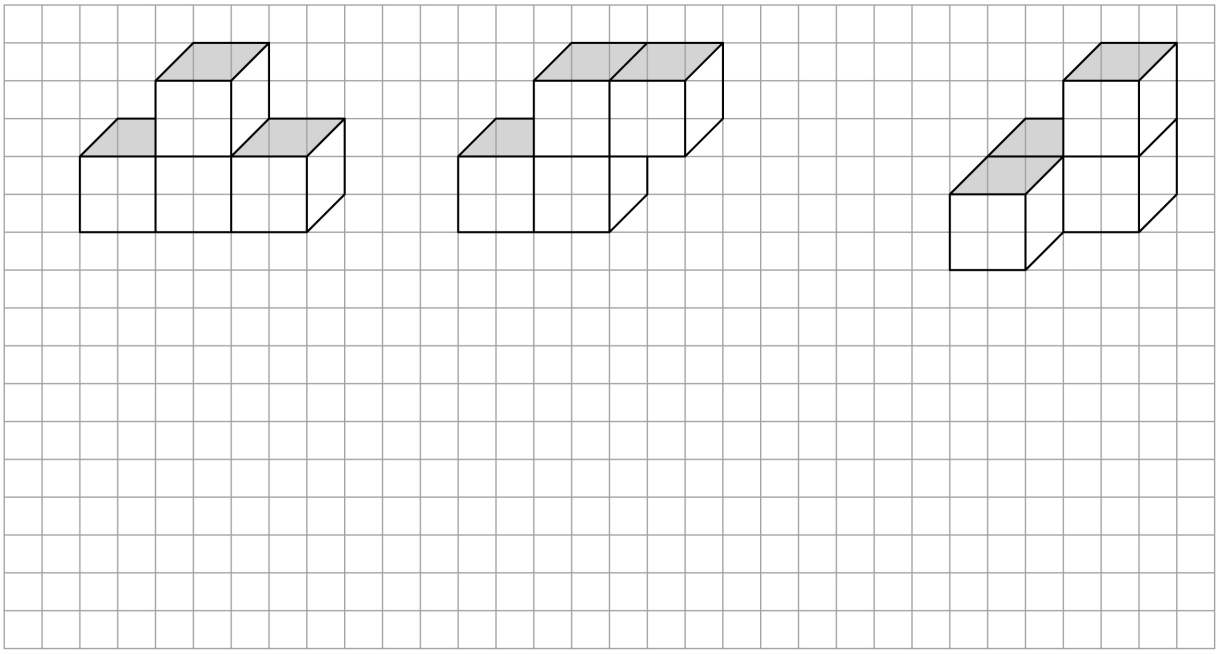
   Give reasons why lining the squares does not work for these quadruples.

Name: Date:

WS 30

Drawing Quadruples

1. Copy these quadruples on the gridded paper.



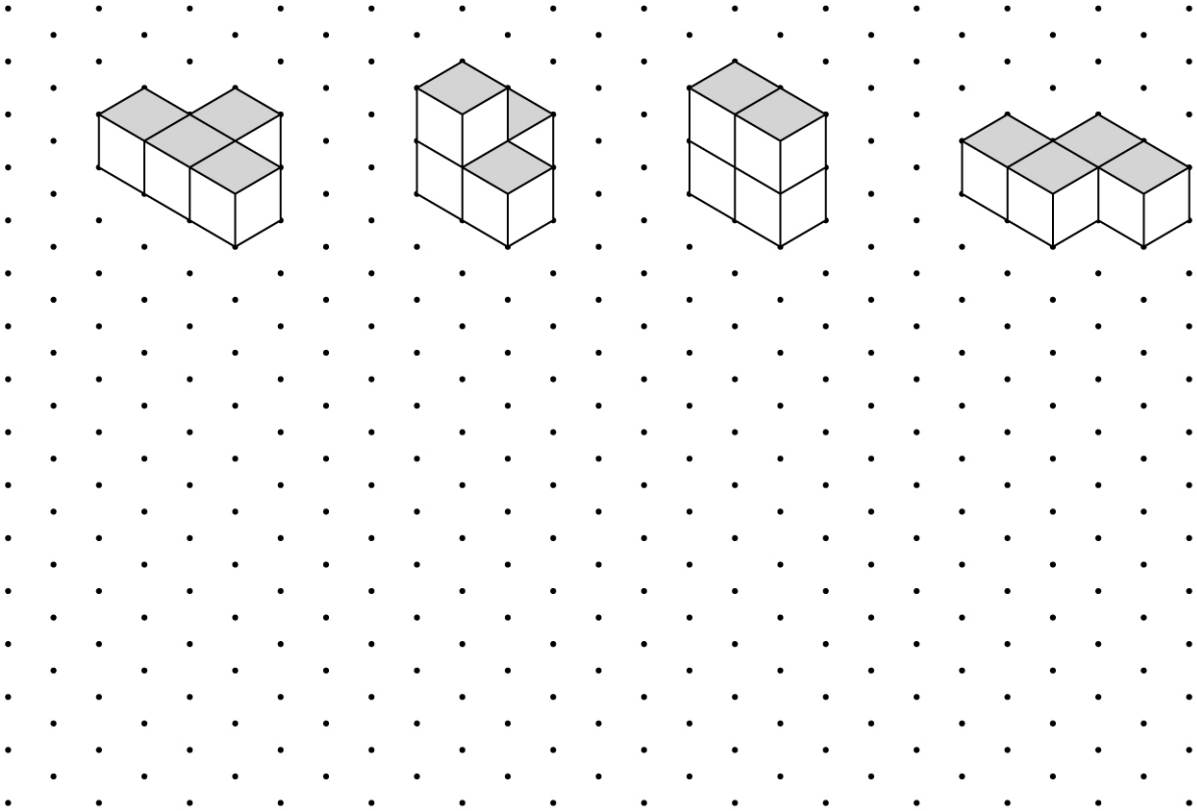
2. Now choose other quadruples and draw them using an oblique perspective.

Name: Date:

WS 31

Drawing Quadruples

1. Copy the quadruples on the dotted grid.



2. Now choose other quadruples and draw them using an oblique perspective.

7 Playing with Cubes
Solid Forms

R. Dolenc-Petz

Reference to Standards and Competences

Spaces and Shapes:
- Orienting in spaces
- Relating two- and three-dimensional displays
- Recognizing, naming and sketching geometric figures

Patterns and Structures:
- Recognizing, describing and extending geometric patterns

Background and Didactic Commentary

With cubes it is possible to conduct a variety of geometric activities. The translation of three-dimensional forms into two-dimensional models enables discoveries in terms of position. By relating three-dimensional and two-dimensional models and working mentally with cubes, nets of cubes and solid cubes, insights into spatial connections and structures become possible. Tasks that require the process of mentally shifting parts within a configuration help to develop children's visualization skills. For this purpose handling of cubes and nets of cubes are particularly suitable, as they also play an important role in the comparison tasks.

Methodological Advice

The tasks in this section stimulate in particular spatial visualization. All the exercises can be done by working with concrete, tangible materials. Children who have difficulties in imagining their actions should work through all the tasks in this chapter by acting manually.

WS 32 is easily completed using a die on the imprinted field of nine squares. The tasks in WS 33 are actively solved by moving a die on large squared paper. For WS 34 six squares are glued together in order to obtain the nets and the corresponding patterns or edges are outlined. The subsequent folding of the nets to obtain cubes and thinking about the different positions prepares pupils for the sketching the possible solutions in the worksheet tasks. Alternatively, a paper cube like that described in the worksheet can be colored and cut up to match for the marking in the nets. By comparing solutions with other children, the pupils can discover different results that can be recorded in the worksheet.

In the cube toss-up (WS 35), the grid can be cut out and glued first, and in so doing the correct cube mappings can be found through tangible activity. Task 2 in WS 35 can be done with a cube that is colorfully painted with appropriate icons. All the tasks in WS 36 can be solved with cubes that are marked according to the worksheet.

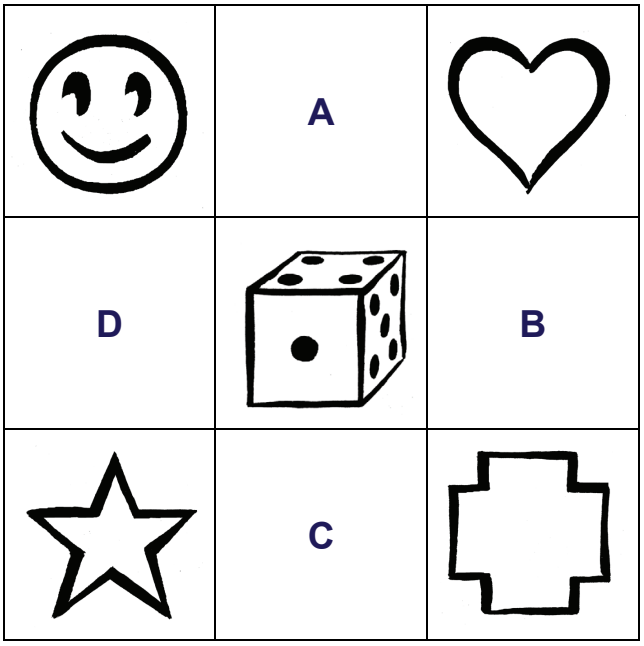
Patterns that the children have invented themselves can be transferred onto squared paper and exchanged with other children, who then can transfer the plans back into real cubes.

Especially important is the comparison of solutions and communication with a learning partner. The translation of the pupil's own mental analysis into language stimulates argumentation skills.

Name: Date:

WS 32

Toppling Dice 1



1. Roll the die mentally. Then check your results with a real die.
   You know that the total of the numbers on opposite faces always equals 7.

   a.  : Topple to A : front ☐ top ☐ right ☐

   b.  : Topple to B : front ☐ top ☐ right ☐

   c.  : Topple to C : front ☐ top ☐ right ☐

   d.  : Topple to D : front ☐ top ☐ right ☐

2. The die in the square of nine is to be rolled onto the fields in the corner using two steps.
   Which numeral then shows on top?

    Option 1: ☐ Option 2: ☐

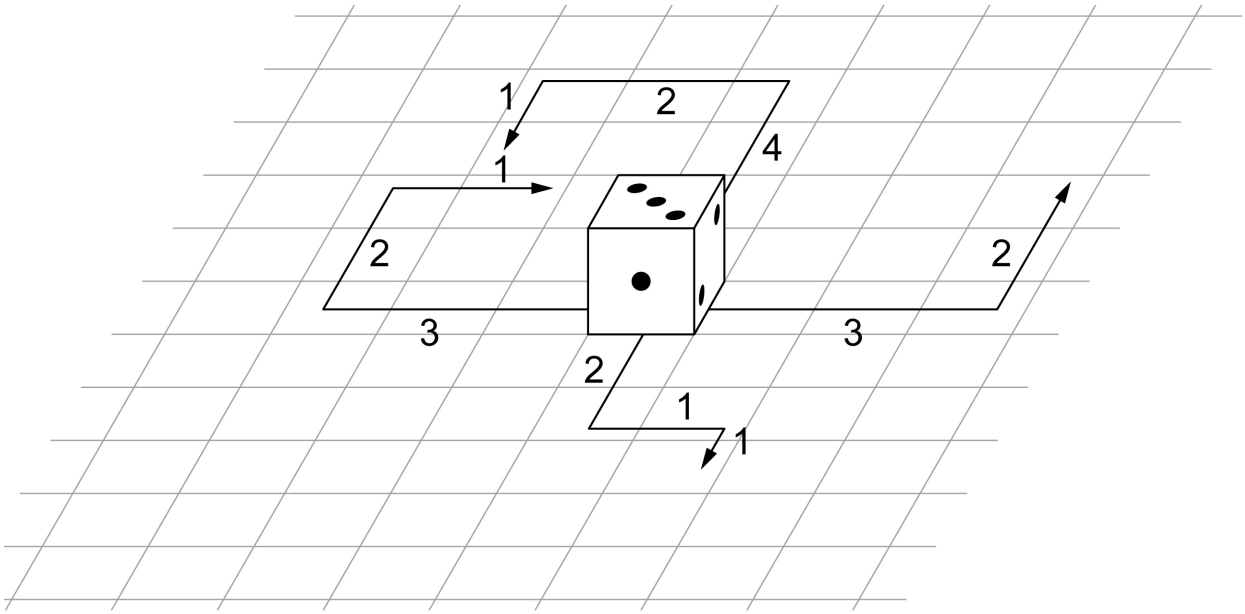
    Option 1: ☐ Option 2: ☐

    Option 1: ☐ Option 2: ☐

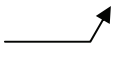
    Option 1: ☐ Option 2: ☐

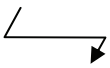
Name: Date:

Toppling Dice 2

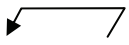


1. Which number is in the front? Tick the correct one.
   You know that the total of the numbers on opposite faces of a die always equals ____.

a.  1 2 3 4 5 6

b.  1 2 3 4 5 6

c.  1 2 3 4 5 6

d.  1 2 3 4 5 6

2. Mark four paths of your own in different colors (red, blue, green and yellow).
   Which number is on top?

red path 1 2 3 4 5 6 blue path 1 2 3 4 5 6

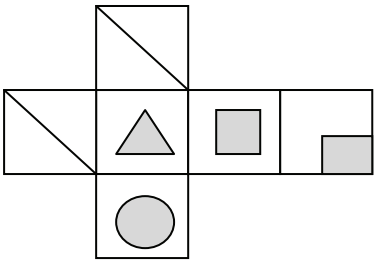
green path 1 2 3 4 5 6 yellow path 1 2 3 4 5 6

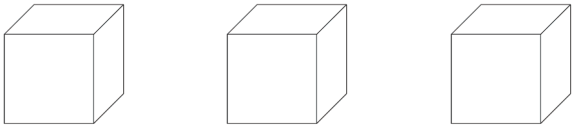
Name: Date:

WS 34

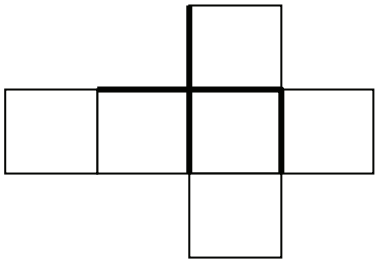
Nets of Cubes

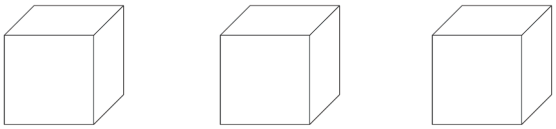
1. Sketch the pattern onto the cubes on the right. Find different possibilities.



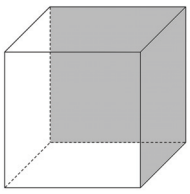


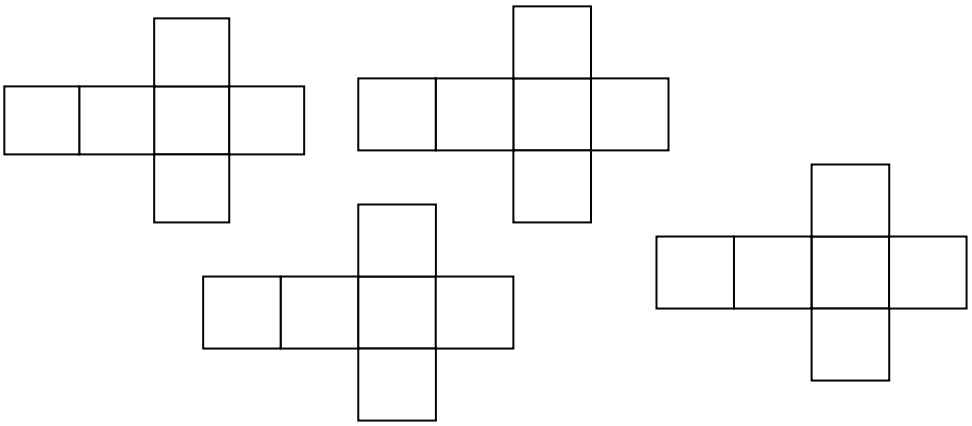
2. Color the edges of the cubes.



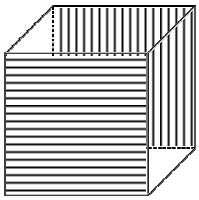


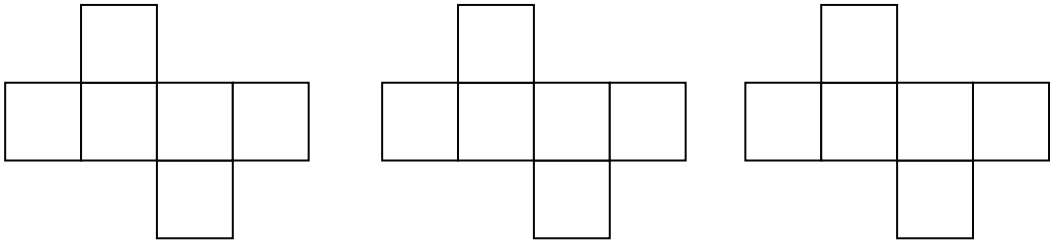
3. Color the nets of the cube, filling in the correct squares.





4. Transfer the patterns from the cube to the nets.

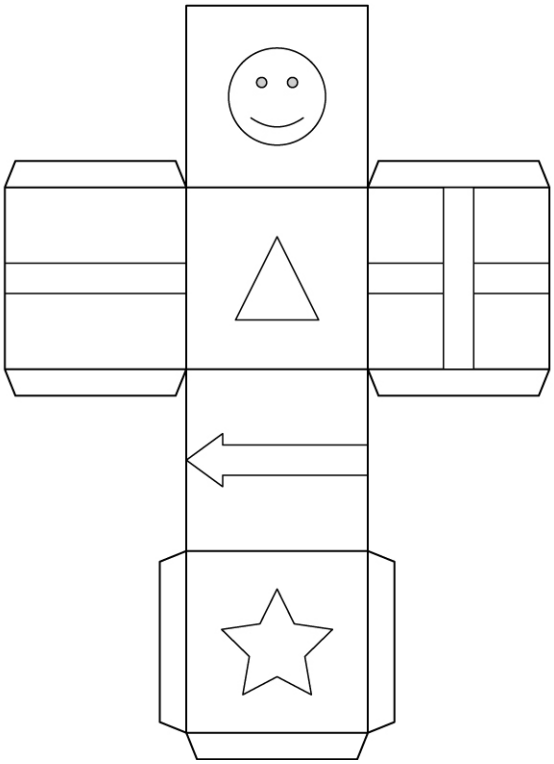




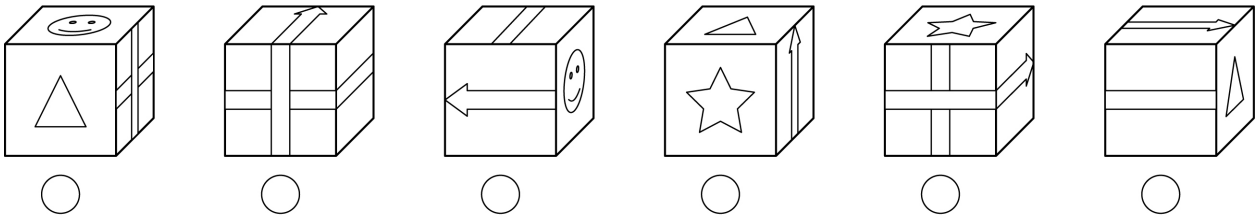
Name: Date:

Cube Toss-Up 1

1. Look at the net carefully. Fold a cube from the net mentally.



2. Which of the depicted cubes match the above net? Tick them.



3. Cut out the net, glue it to make a cube and check.

4. Here you see the same cube four times.
   Which icon does each of them have on the bottom?

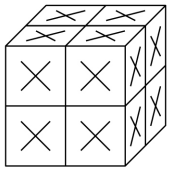
Bottom:

Name: Date:

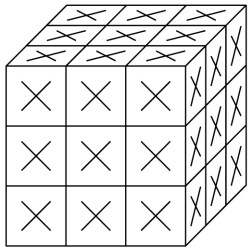
WS 36

Cube Toss-Up 2

These big cubes are assembled from small cubes.



Cube of Two



Cube of Three

1. How many small cubes are in:

   the cube of two: _____? the cube of three: _____?

2. Recreate the big cubes and mark each small cube in pencil with an “x” on the outside.

3. Write down the correct number.

	Cube of Two	Cube of Three	Cube of Four
cubes without a cross	_____	_____	_____
cubes with 1 cross	_____	_____	_____
cubes with 2 crosses	_____	_____	_____
cubes with 3 crosses	_____	_____	_____
cubes with 4 crosses	_____	_____	_____
check: cubes in total	_____	_____	_____

4. Transforming cubes

   a) Transform your cube of two so that
   - only one cross on each big surface can be seen.
   - the same pattern on each big surface emerges.

   b) Transform your cube of three so that these patterns are on each big surface. Invent your own patterns and sketch them on gridded paper.

8 The Geometric Board
Plane Figures

H. Gutmann

Reference to Standards and Competences

Recognizing, naming and depicting geometric figures:

- Arranging plane figures by characteristics and matching the figures with mathematical terms
- Making models from plane figures and analyzing them
- Depicting simple figures in grids

Comparing and measuring surfaces and volumes

- Measuring and comparing surface areas of plane figures by disassembling, measuring the figures by lining with unit surfaces
- Analyzing perimeters and surface areas of plane figures

Background and Didactic Commentary

The geometric board is a didactically-multifaceted, useful tool for classroom activities. It provides many possibilities, like stimulating spatial awareness, displaying different plane figures, carrying out difficult geometrical analyses or identifying surface areas and perimeters. The special structure of the geometric board (equal gaps between the bars) enables pupils to comprehend an area and a figure's dimensions and to compare these characteristics with those of other forms. Working with the geometric board allows space for children to explore on their own and make new discoveries. Not only do pupils practice working with perimeters and surface areas, but the geometric board also helps develop the pupils' abilities as follows:

- The coordination between eyes and hands is stimulated by stretching the rubber bands around the nails.
- The size of the geometric board accommodates children with impaired motor skills.
- The stretched figure stays in full view and can be copied and worked with at the pupil's own pace.
- Descriptions of positions like right/left, top/bottom, etc. can be practiced when describing the surface.
- The perimeter can be described with terms like round, around, outside or similar.
- The children have to look at the surfaces accurately and recreate or trace them.

The active engagement with surface areas is an important requirement for understanding characteristics and calculations relating to the plane figures. Another important aspect is the verbalization of the pupils' findings. Exchanging ideas with a partner, trying to give comprehensible explanations as well as drawing conclusions is indispensable for the understanding of mathematical facts. Many schools provide geometric boards, though with a different number of rows of nails. A board with 25 nails (5 x 5) is a good choice for the lessons as it allows various forms. Therefore the worksheets use boards with 5 x 5 nails.

For making your own geometric boards detailed instructions are provided in the enclosed CD.

Methodological Advice

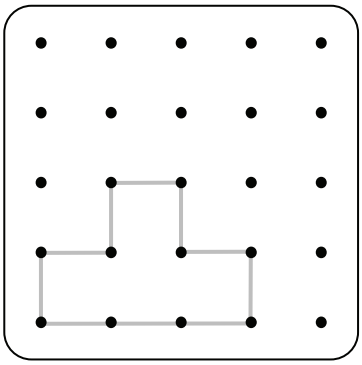
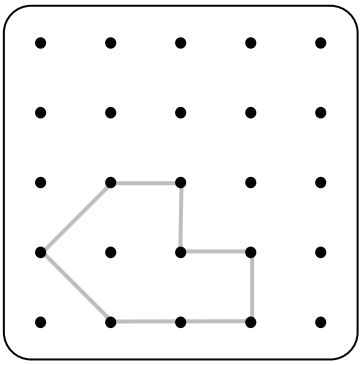
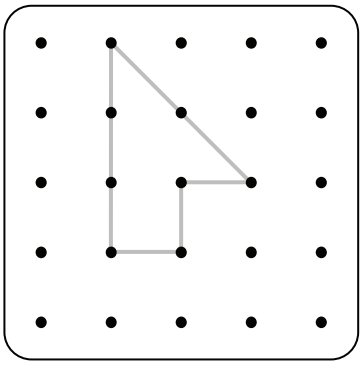
Before starting with specific tasks the children must be allowed to make their own discoveries first. Here they learn how to tighten the rubber band in order to let a figure pop up. Through trial-and-error they invent pictures and surfaces that they may not have consciously created before. First the children create mostly objective pictures (a boat, house ...), which can be sketched onto original-sized geometric boards made of paper (see CD). Only after this creative period it is useful to start with concrete exercises.

Name: Date:

WS 37

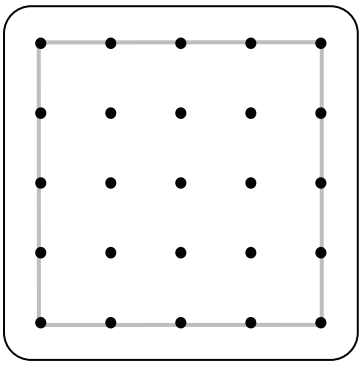
Creating Two-Dimensional Figures

1. Create these figures on your board.

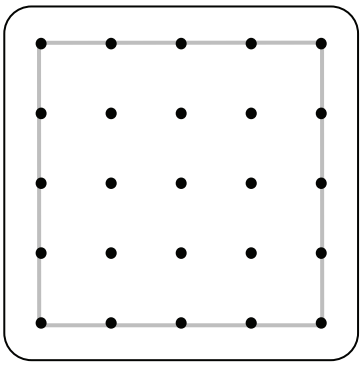
2. Create your own figures on your board. Your partner copies them.

3. Create this square.

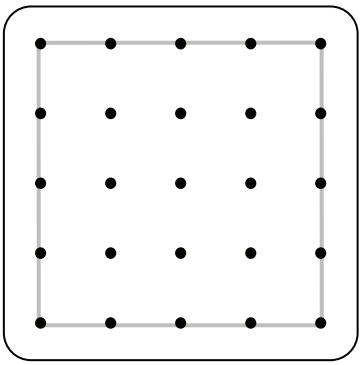


4. Take a second and third rubber band. From the square, make:

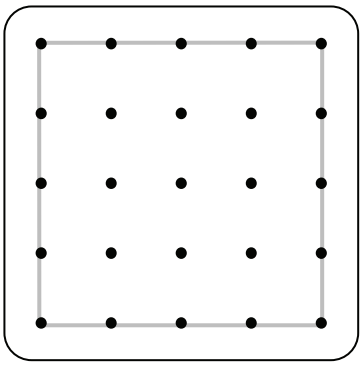
4 squares



4 triangles



4 rectangles



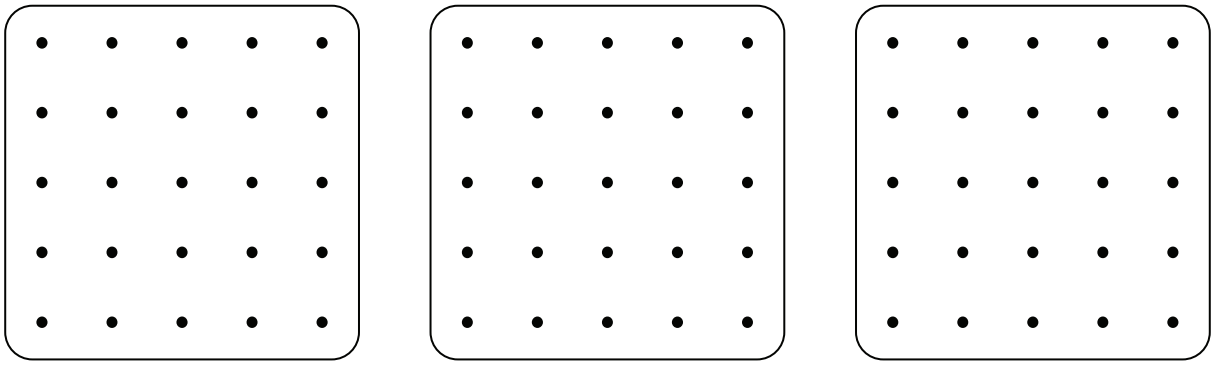
Sketch the additional rubber bands into the above squares.

Name: Date:

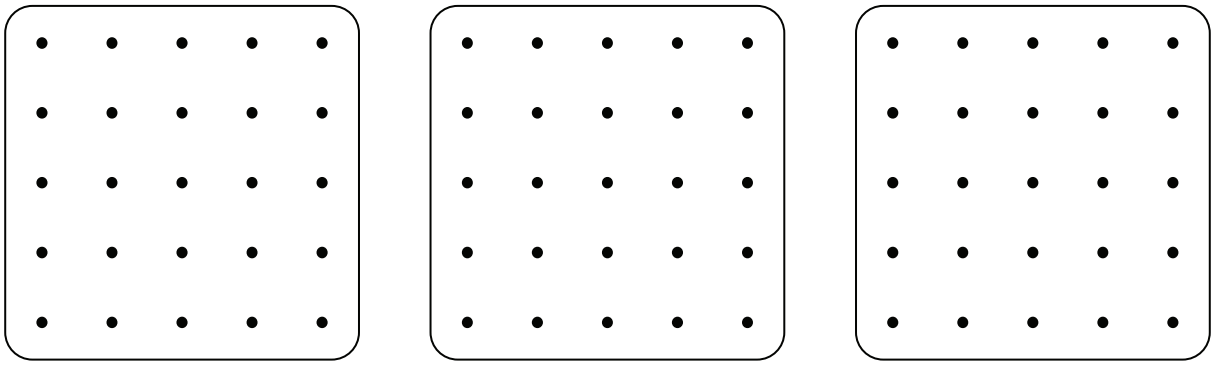
WS 38

Creating Squares

1. Create squares of different sizes. How many different ones do you find?
   Sketch them in different colors.

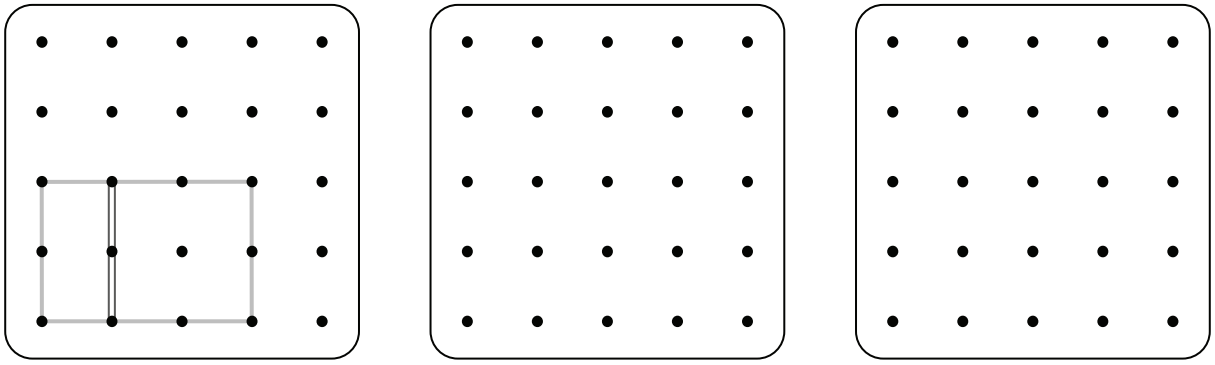


2. Create different rectangles. How many do you find now?
   Sketch them in different colors.



What do you notice? _____

3. Change a rectangle:
   Use another rubber band and sketch it. Write down what comes out.



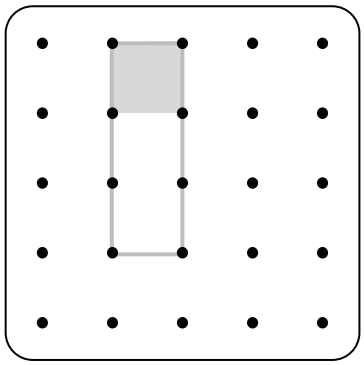
_____ _____ _____

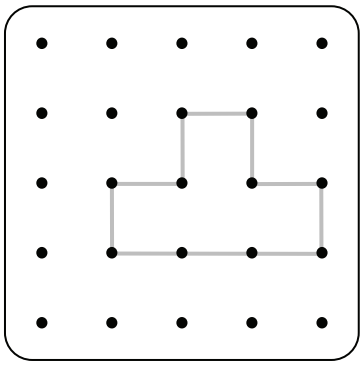
Name: Date:

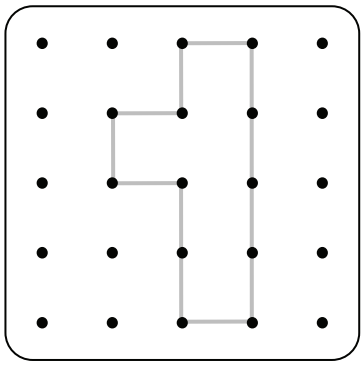
WS 39

Measuring Surface Areas 1

1. You can measure the size of a figure with the help of small squares □. The size of a two-dimensional region is also called surface area. How many □ fit into each figure?



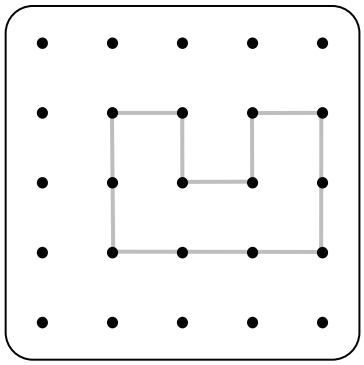




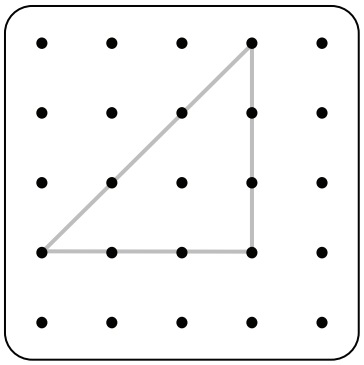
_____ □ _____ □ _____ □

2. Now create the figures for yourself. Think about how many small squares and triangles fit into them. Record the number in the table below.

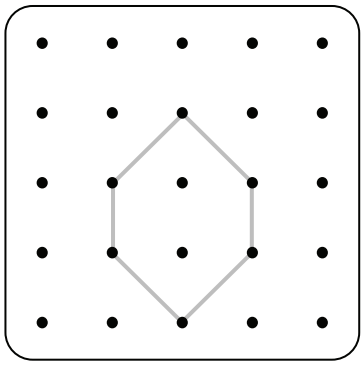
A



B

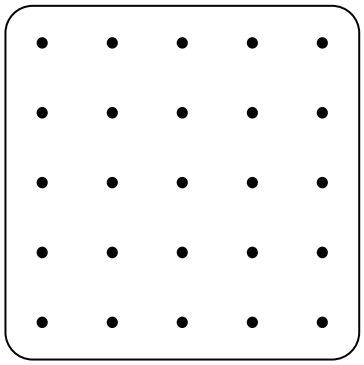


C



	□	◿
A		
B		
C		
D	2	6

Create and sketch the figure D.

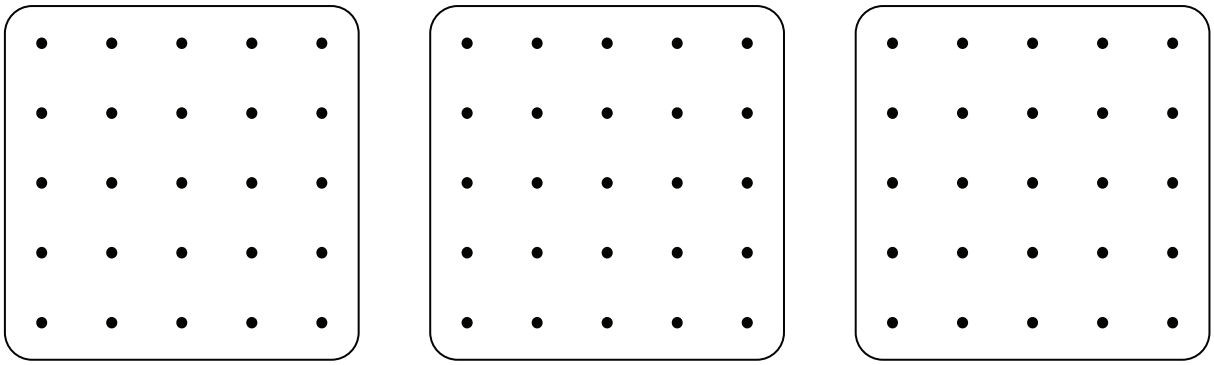


Name: Date:

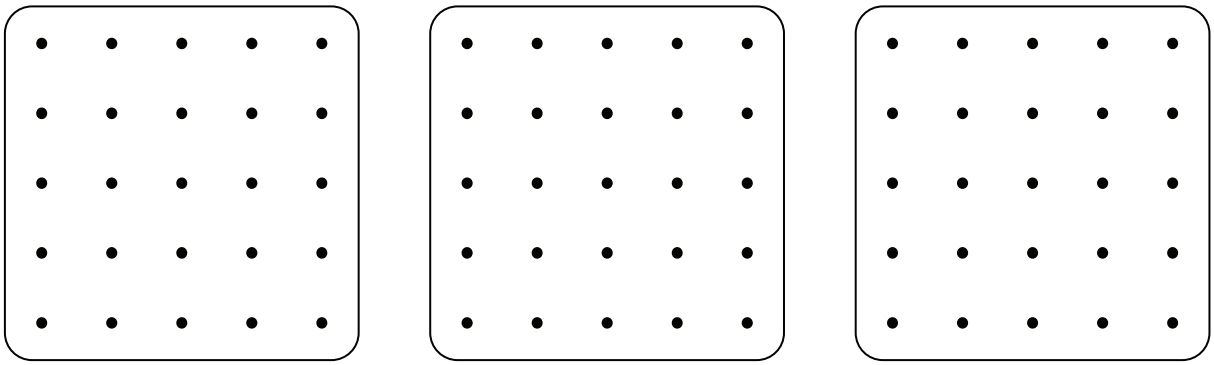
WS 40

Measuring Surface Areas 2

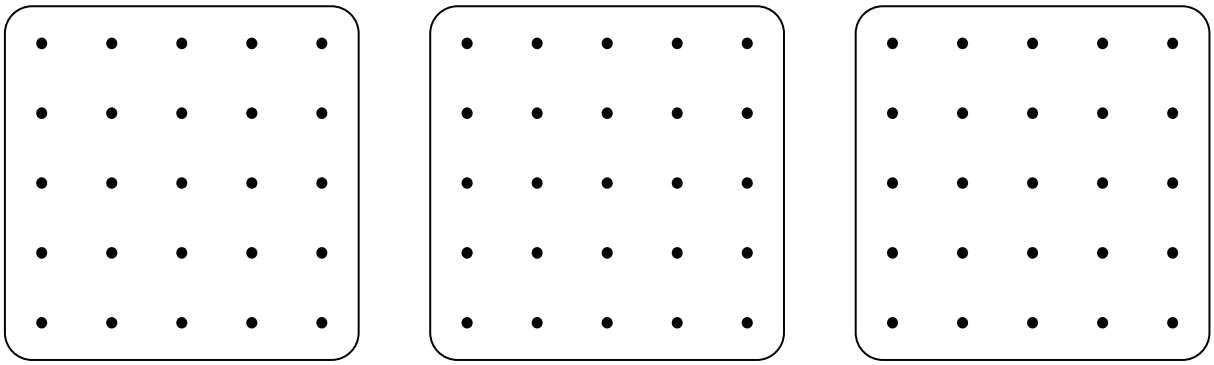
1. Tighten figures with 3 □ . Sketch them in different colors.



2. Find figures with a surface area of 4 □. Sketch them.



3. Choose a size for your surface area: ____ □. Tighten different figures with this surface area. Sketch them.



How did you do it?

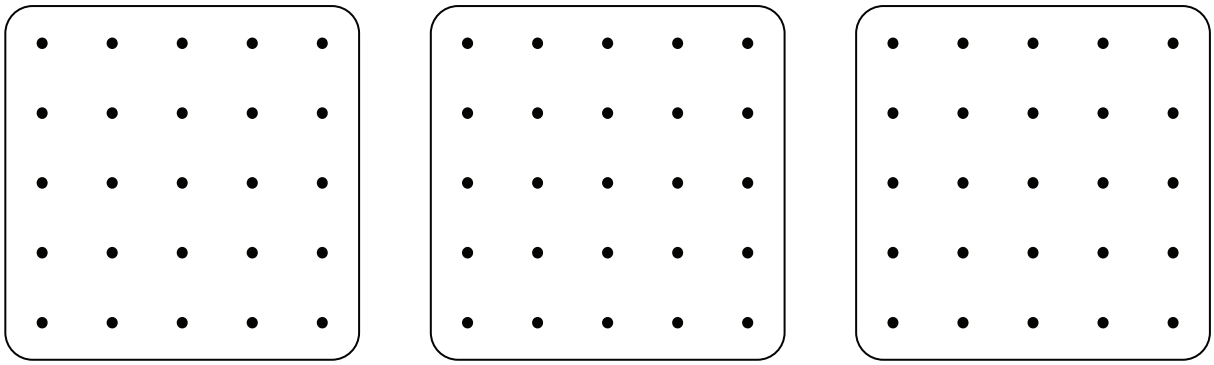
_____

Name: Date:

WS 41

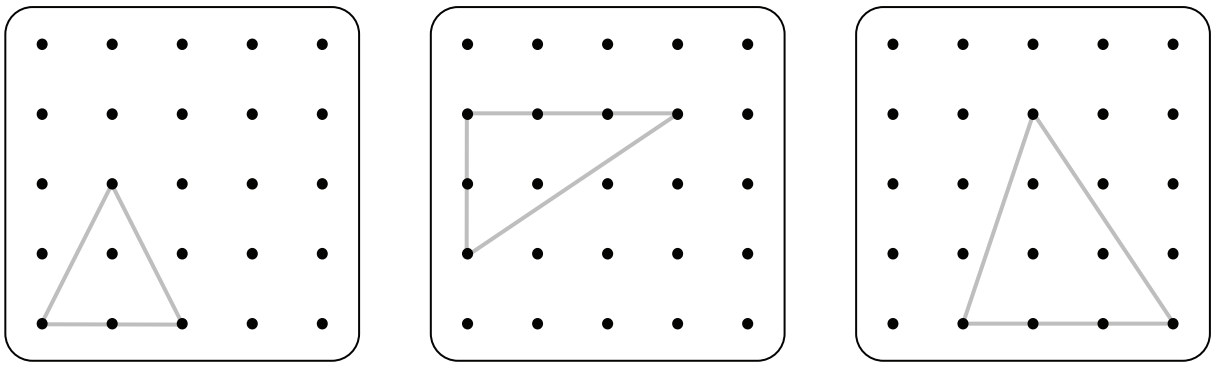
Measuring Surface Areas 3

1. Create different triangles with a rubber band on your geometric board.
   Sketch them in different colors.



What do you notice?

2. Which triangle is the biggest?



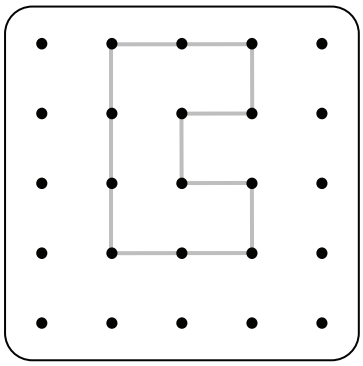
Explain your decision.

Name: Date:

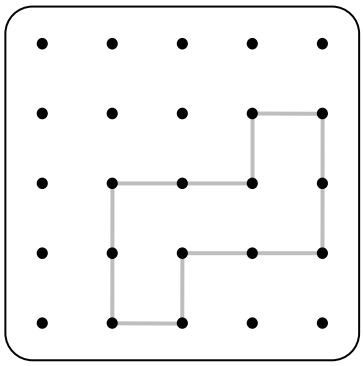
WS 42

Finding Perimeters

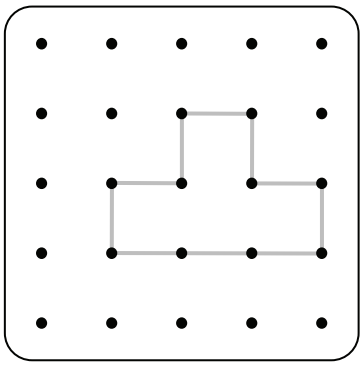
1. The perimeter of each figure on the geometric board can also be defined. The tightened rubber band is the edge of the surface. The length of the edge is called perimeter. The length of the rubber band from one nail to the next (not diagonally!) will be the measuring unit for the perimeter.



_____ units



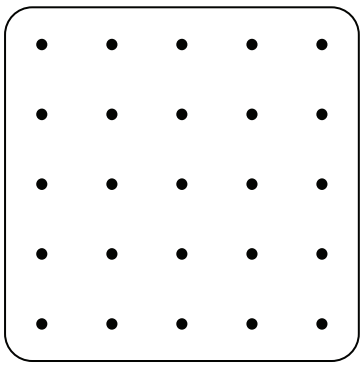
_____ units

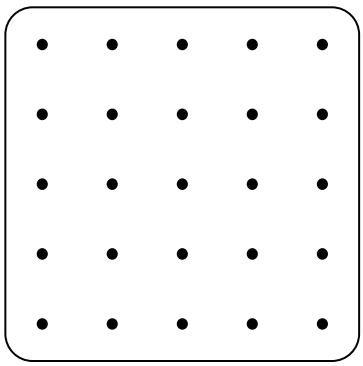


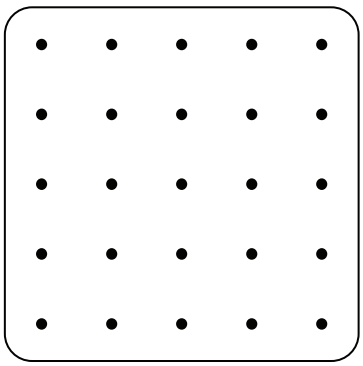
_____ units

2. Which of the above figures has the biggest perimeter? Color it.

3. Create different figures that have the **same** perimeter. Sketch them.







Describe your approach.

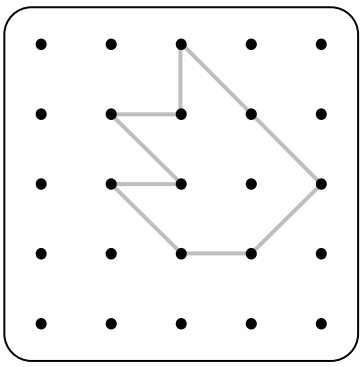
Name: Date:

WS 43

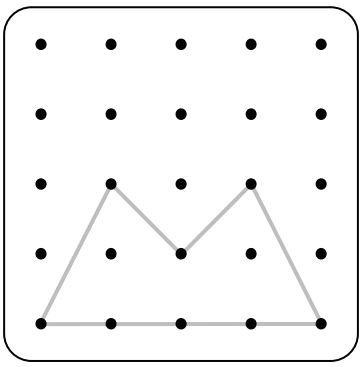
Puzzling Over the Geometric Board

Tim and Lea have created the following figures.

Tim



Lea



“My figure has a bigger surface area than yours,” says Tim. Is he right?

Yes: ☐ No: ☐

What did you find out? Describe and explain what you learnt.

Tim also says, “Besides, my figure has a bigger perimeter than yours.”
Is this statement correct?

Yes: ☐ No: ☐

Explain your opinion.

9 The Drawing Clock

Plane Figures – Sketching Regular Polygons and Patterns

R. Dolenc-Petz

Reference to Standards and Competences

Spaces and Shapes:

- Recognizing, naming and depicting geometric figures
- Making drawings with aids

Patterns and Structures:

- Recognizing, naming and depicting regularities in geometrical and arithmetical patterns
- Developing, describing and methodically altering arithmetical and geometrical patterns.

Background and Didactic Commentary

Clocks and clock faces are known to children from their living environment. A drawing clock is an aid that consists of a circumference that holds 60 marks, just like a clock. Every fifth scale (marking hours/five-minute-segments) is highlighted. This way, regular polygons can be sketched and deduced easily. With a ruler, two points on the circumference are connected by chords of equal length at a time. As the number of corners corresponds to the factors of 60, regular triangles, squares, pentagons, hexagons, decagons and dodecagons emerge. In addition, 20-, 30-, and 60-gons are possible. These are not easy to draw, and the corners are difficult to recognize. The combination of different polygons within a drawing clock reveals interesting patterns that the pupils can color.

With the aid of the figurative display of hours in the drawing clock, the emerged figures can be described by numeral patterns – a square, for example, could correspond to marking 12, 3, 6, 9 o'clock – while, in reverse, numeral patterns can be translated into figures. This way the connection of geometry and arithmetic is demonstrated.

Methodological Advice

A vivid and activity-oriented introduction to the topic is the "hour clock", a board with 12 nails that mark the hours. With a string or rubber bands different figures or patterns are displayed. For sketching these polygons on WS 44, the teacher must explain to the pupils that the drawing clocks in the worksheets show not only the hours but also minutes. Furthermore, the bold marks on the drawing clocks represent the hours.

It has proved successful to discuss with the children in advance that the corners of the sketched polygons have to sit exactly on the markings. Advise the pupils to mark where the corners are lightly before sketching the figure and then connect these marks with the aid of a ruler. This requires and trains the children in meticulous work.

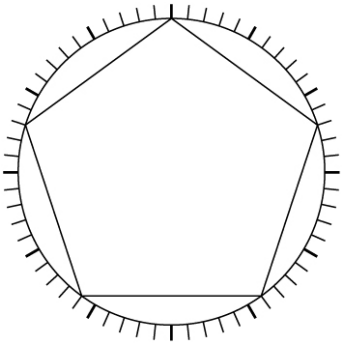
On the „hour clock" triangles, squares, hexagons and dodecagons can be created. For the side-length of a 15-gon, hour-markings are required as well as minute-markings. The connectedness between possible polygons and the factors of 60 certainly does not become accessible to all children by itself and has to be explicitly explained.

Further tasks with the drawing clock (WS 45 – 48) are open to differentiation and offer a wide variety of design possibilities. The results found by pupils provide motivation for discussions, e.g. about figures that have come up through the combination of different basic figures in the patterns.

Suggestion: Squares and hexagons parquets can be laid and glued on using cut-out, colored triangles.

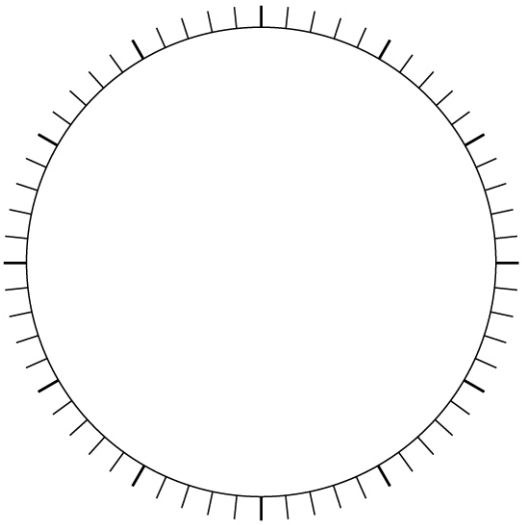
Name: Date:

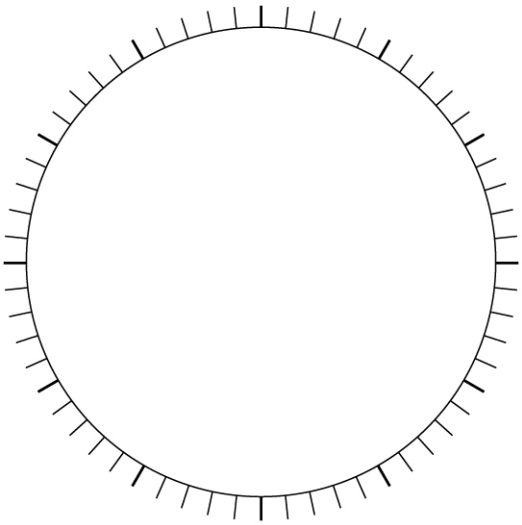
Drawing Regular Polygons

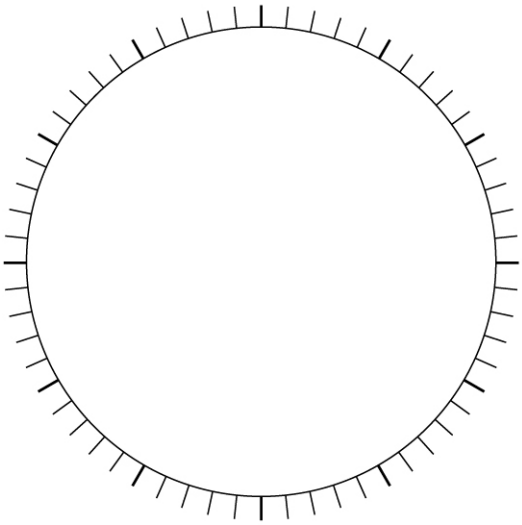


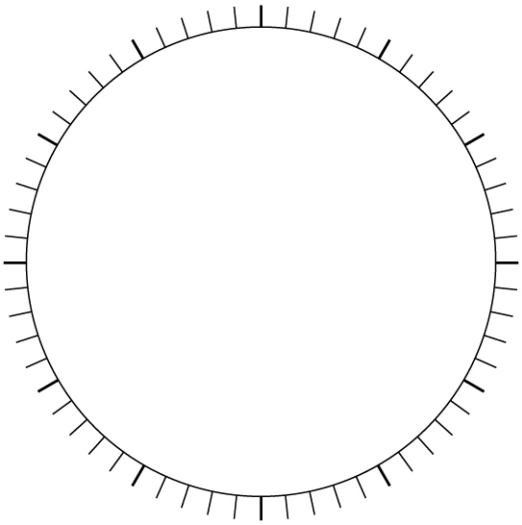
A drawing clock has 60 marks – just like a clock face.
With a drawing clock you can sketch regular polygons accurately.

1. Draw regular polygons into the drawing clocks.
   - First mark the corner points and then connect them with the ruler.
   - Color in the polygons.





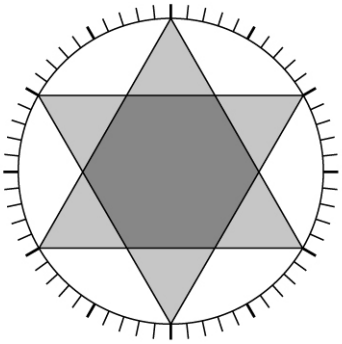


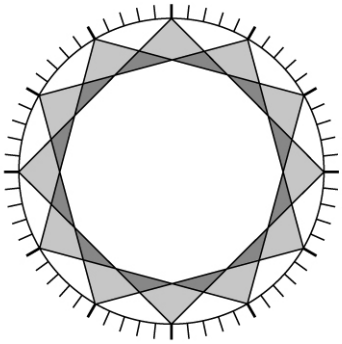


Name: Date:

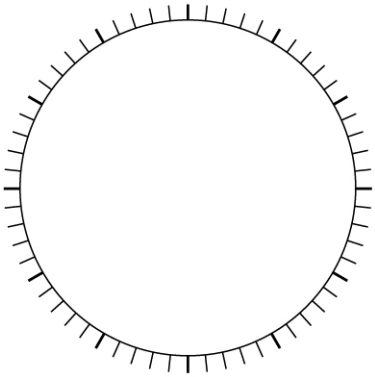
Patterns from Polygons

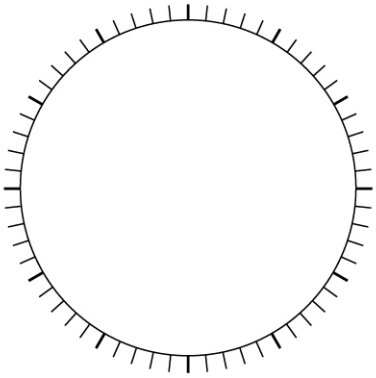
With the drawing clock you can also draw nice patterns.

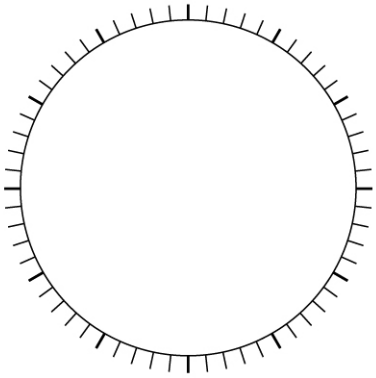




1. Draw patterns with identical polygons. Color them in.

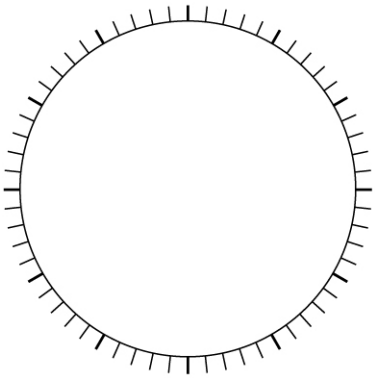


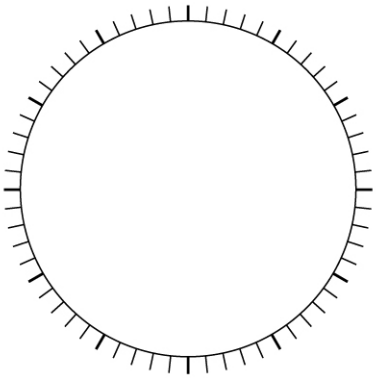


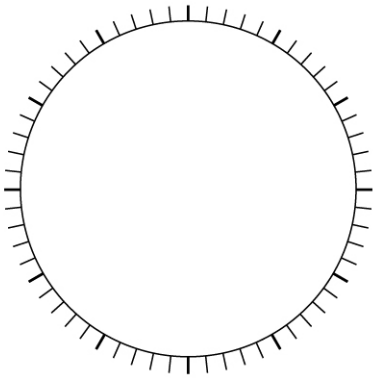


Which forms do you discover in your patterns?

2. Draw patterns with different polygons. Color them in.





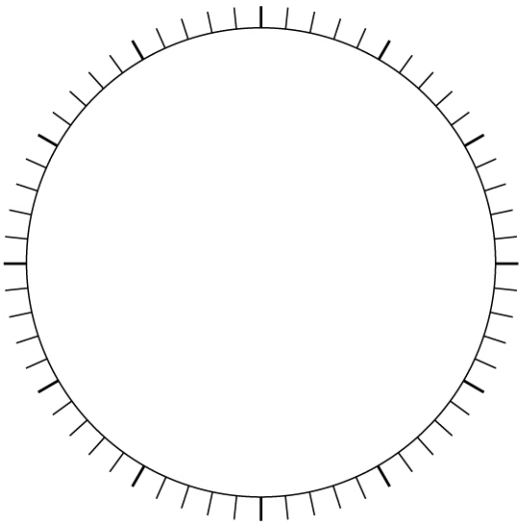


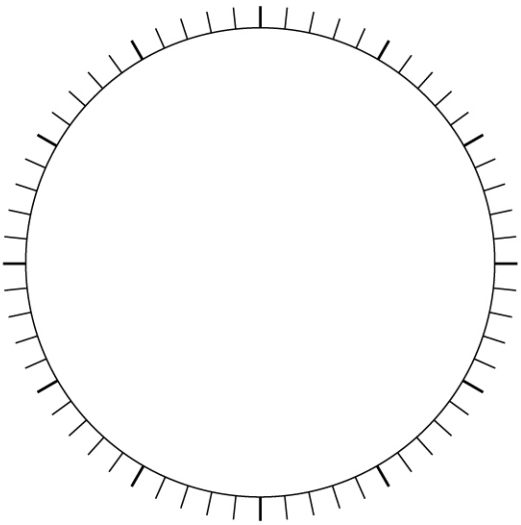
Which forms do you discover in your patterns?

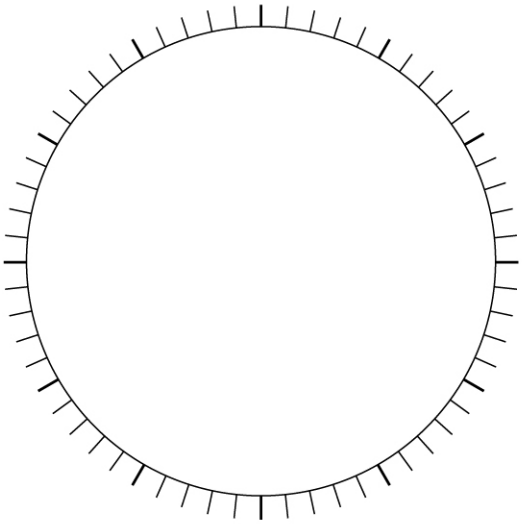
Name: Date:

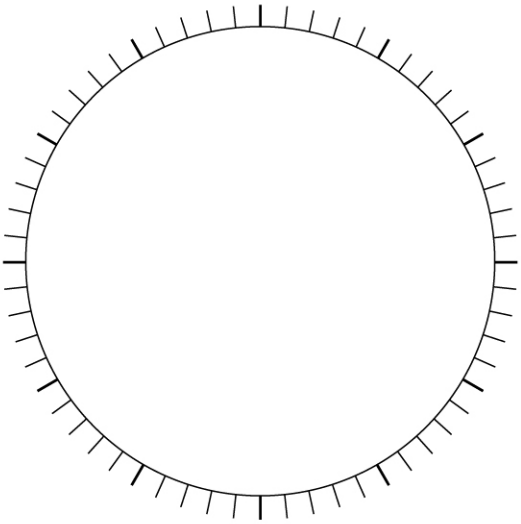
Inventing Patterns

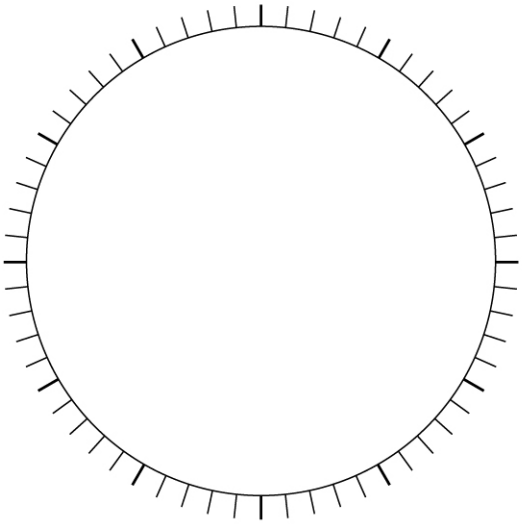
Invent your own patterns. Color them in.

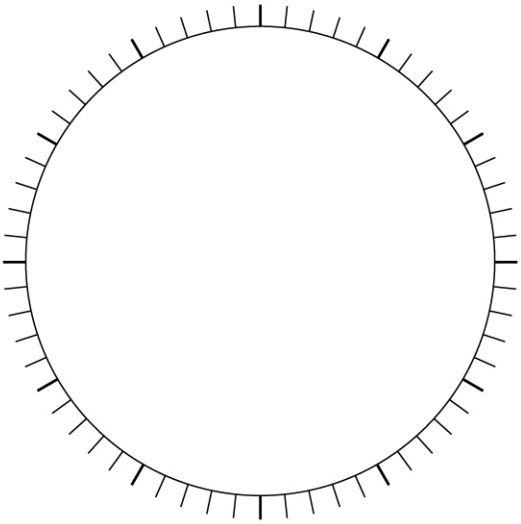












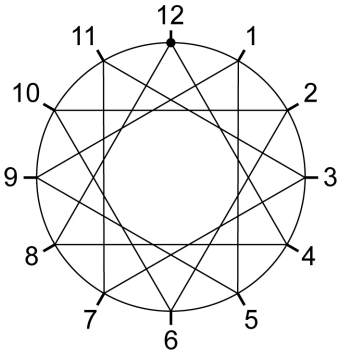
Name: Date:

WS 47

Image Patterns – Number Patterns

1. Write down the matching number pattern for each image pattern.
   What do you notice?

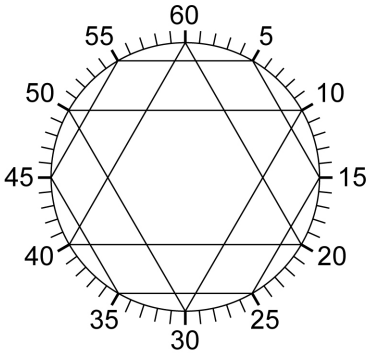
a)



12 – 4 – 8 – 12

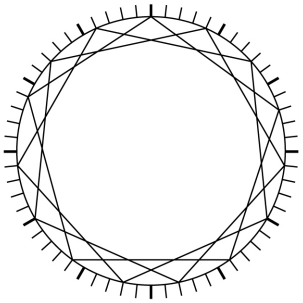
1 – 5 –

b)



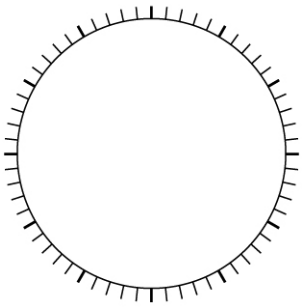
55 – 5 –

c)



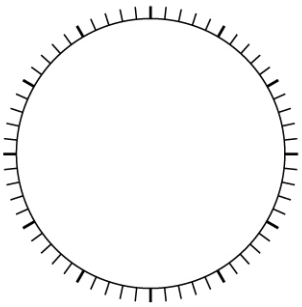
2. Draw your own image pattern. Write down the matching number pattern.

a)



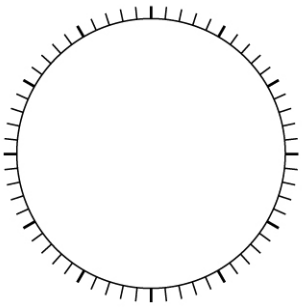
number pattern for a)

b)



number pattern for b)

c)



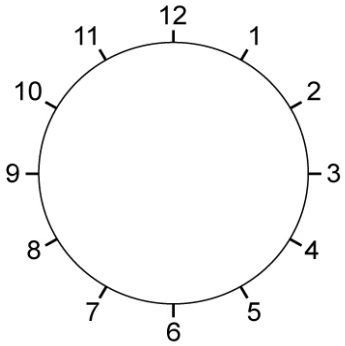
number pattern for c)

Name: Date:

Number Pattern – Image Pattern

Read the number patterns.

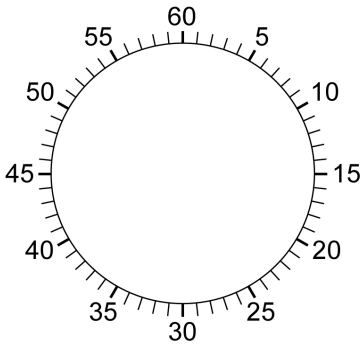
- First write down of which figures the image pattern consists.
- Then draw in the image patterns.



Number patterns	figures in the pattern
12 – 2 – 4 – 6 – 8 – 10 – 12	
1 – 9 – 5 – 1	
11 – 3 – 7 – 11	

What do you notice? _____

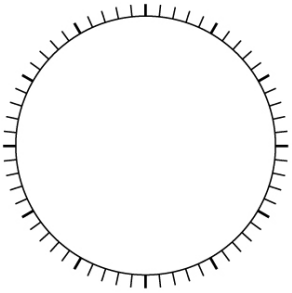
_____



Number patterns	figures in the pattern
6 – 21 – 36 – 51 – 6	
56 – 11 – 26 – 41 – 56	
1 – 21 – 41 – 1	
51 – 11 – 31 – 51	

What do you notice? _____

_____



Number patterns	figures in the pattern
5 – 17 – 29 – 41 – 53 – 5	
9 – 21 – 33 – 45 – 57 – 9	
13 – 25 – 37 – 49 – 1 – 13	

What do you notice? _____

_____

10 Exercises with Matches
Plane Figures

P. Ihn-Huber

Reference to Standards and Competences

Spaces and Shapes – Recognizing, naming and depicting geometric figures

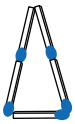
- Arranging forms and plane figures by characteristics and matching these characteristics and figures with mathematical terms
- Making and analyzing models from plane figures (building, placing, disarranging, assembling, …)

Spaces and Shapes – Comparing and measuring surface areas and volumes

- Measuring and comparing surface areas of plane figures by lining the figures with unit surfaces
- Analyzing perimeters and surface areas of plane figures

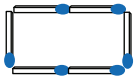
Background and Didactic Commentary

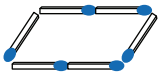
Matches are perfectly suitable due to their uniform length for creating polygons with equal, "countable" side lengths. Initially the children create different figures with the matches, and, among others, the following figures emerge:

triangle 

equilateral triangle 

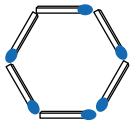
square 

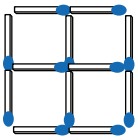
rectangle 

parallelogram 

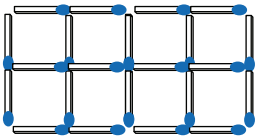
rhombus 

pentagon 

hexagon etc. 

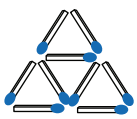


By using more matches, all rectangles can be divided up into small standard squares, and the area can be determined. In so doing, the area of four standard squares can be visualized by adding the inside matches of a 2x2 square (see left) or the area of eight standard squares as a 2x4 rectangle (see below).
When creating different rectangles with a fixed number of matches and then observing their surface areas, a connectedness between perimeters and surface areas becomes evident.



Equilateral triangles can also be used as standard triangles to determine areas, and so the smallest possible equilateral triangle becomes a unit. This unit is used four times in the triangle with the side length 2, nine times in the triangle with the side length 3, etc.
The smallest possible triangle area can simply be defined as unit 1.

Toss ups train the eye for geometrical figures and connectedness.
They can also – without material, only in the imagination – be used for mental exercises. This, however, is only to be accomplished by the children after an extensive time of practicing.



Methodological Advice

The first task (WS 49) serves to approach the material freely. The children place figures with their matches. At group tables it is easy to create a little exhibition: The children place matching figures according to word cards for different plane figures which they can even write themselves. Here it can be required that not any two figures are allowed to be the same. Naming the parts of complicated figures trains the cognitive ability and can be done in partner work. Placing at somebody's command can be continued as partner work, it even puts itself forward to let the children write their instructions on note cards.

The methodical handling with different-sized squares (WS 50) doesn't only train geometric thinking but also sharpens the eye for patterns and structures. Therefore it seems important to compare the various squares in terms of their perimeters (= number of matches, thus side length times 4) and their surface (= number of standard squares, thus sidelength^2).

On WS 51 different rectangles are placed with a given number of matches and the surfaces are considered. After discussing the task in class the children can work at it independently, first with given numbers, later with numbers of their choice. Here findings are worded: "With odd numbers you can't make a rectangle." "Only when the number can be divided by 4 you can place a square." etc. Considering surfaces in comparison reveal that the biggest surface of the same perimeter is always the closest possible to the square.

Also with triangles regularities are identified in terms of side length, number of used matches and surface area (WS 52).

The toss-up tasks (WSs 53 and 54) are designed for trying independently first, although sometimes it can be interesting to compare solutions between the pupils. These tasks can also be completed by self-invented exercises. It is particularly pretty when the information is glued on the cards with real matches (solutions are sketched on the back of the card).

Name: Date:

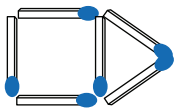
WS 49

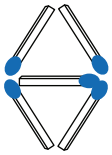
Placing Figures with Matches

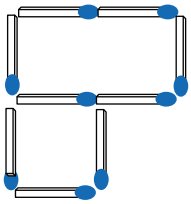
1. Get a box of matches and place different figures.

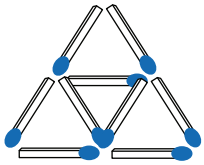
square | triangle | pentagon | house | rectangle | hexagon | star

2. Copy these figures and describe what you have placed.









3. Place these figures as instructed. Then sketch them.

Make a square from 1 match edge length. Place a triangle from two matches against the upper edge.

Make a rectangle by height 2 and by width 4. Place a square from edge length 1 to the right bottom corner.

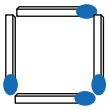
Make a square from 1 match edge length. Place against each edge a triangle with 2 matches.

Make a rectangle by width 2 and height 1.
Place against each of the top matches a triangle with two matches each.

Name: Date:

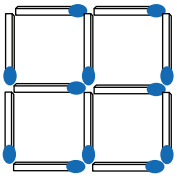
Placing Different Squares

1. Make squares of different sizes.
   How many matches do you need apiece?



Side length	1	2	3	4	5		…	10	…	20
Number of matches	4									

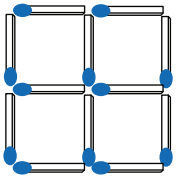
2. Each bigger square can be divided into smaller squares by placing matches. How many small squares fit into each big square?
   We say then, for example, the square with the side length 2 has the surface area 4.
   Try it. Draw up a table as well:



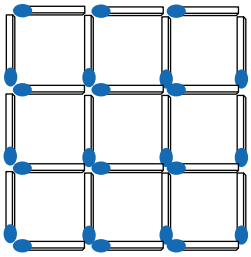
Side length	1	2	3	4	5		…	10	…	20
Surface area	1	4								

3. Start with the smallest square.

Now add matches to make the next bigger square. Think:
How many matches do you need?
By how many squares does the surface area increase?



Again add matches to make the next bigger square. Think:
How many matches do you need?
By how many squares does the surface area increase?



Think about how the pattern continues. To do so complete the table and consider how many matches you would need for a square with the side length 10.

Side length	1	2	3	4	5		…	10
Surface	1	4						
New matches	4	8	12					
Matches in total	4	12	24					

Name: Date:

WS 51

Placing Rectangles

1. Take 20 matches.
   a. Try and place different rectangles with them.
   b. All rectangles have the same perimeter. But do they also have the same surface area? Consider and write down. Mark the smallest and the biggest surface area.

Length	6							
Width	4							
Surface	24							

2. Now take 24 matches. Lay different rectangles. Draw up a table for this as well. Mark the smallest and the biggest surface area.

Length	2							
Width	10							
Surface	20							

3. Now take 12 matches. Lay different rectangles. Draw up a table as well. Mark the smallest and the biggest surface area.

Length	4							
Width	2							
Surface	8							

4. Choose any number of matches and make rectangles from them. What do you notice?

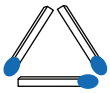
5. Jonathan and Catherine have an enclosure for their guinea pigs in the garden. It is made from 8 equal pieces of fence that are stuck into the grass. Consider how they have to put up the parts so that the guinea pigs have the biggest possible enclosure. Draw up a plan and explain your answer.

Name: Date:

WS 52

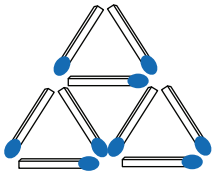
Placing Different Triangles

1. Make triangles of different sizes. How many matches do you need apiece? Complement the table.



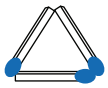
Side length	1	2	3	4	5		…	10	…	20
Number of matches	3									

2. Each bigger triangle can be divided into smaller triangles by placing matches. How many small triangles fit into each big triangle? We say then, for example, the triangle with the side length 2 has the surface area 4.
Try it. Draw up a table as well:

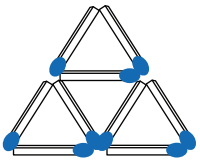


Side length	1	2	3	4	5		…	10	…	20
Surface area	1									

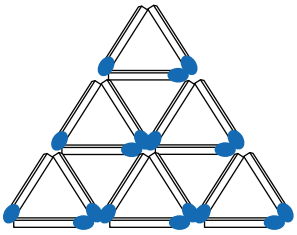
3. Start with the smallest triangle.



Now add matches to make the next bigger triangle.
Think:
How many matches do you have to use?
By how many triangles does the surface area increase?



Again add matches to make the next bigger triangle.
Think:
How many matches do you have to use?
By how many triangles does the surface area increase?



Think about how the pattern continues. To do so complete the table and consider how many matches you would need for a triangle with side length 10.

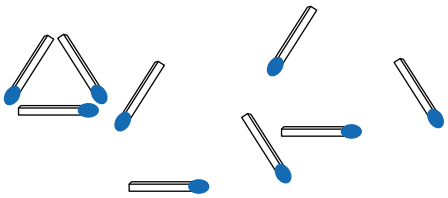
Side length	1	2	3	4	5		…	10
Surface	1	4						
New matches	3	6	9					
Matches in total	3	9	18					

Name: Date:

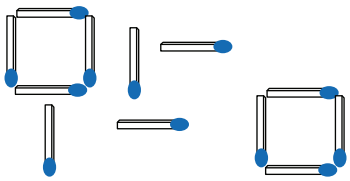
WS 53

Toss-up Tasks 1

1. Make from these 9 matches:
   a. 1 triangle
   b. 2 triangles
   c. 3 triangles
   d. 4 triangles
   e. 5 triangles



2. Make from these 12 matches:
   a. 1 square
   b. 2 squares
   c. 3 squares
   d. 4 squares
   e. 5 squares

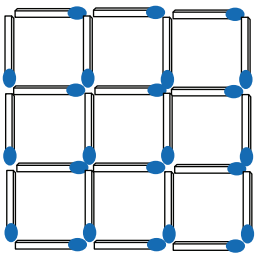


3. Make 3 squares of equal size from 10 matches.

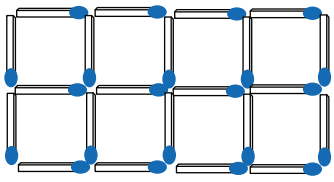
4. Make 3 squares of equal size from 11 matches.

5. Make 4 rectangles of equal size from 9 matches.

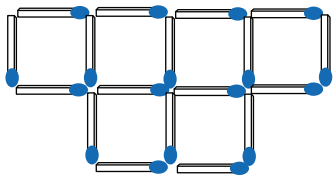
6. From this 24 matches …
   a. remove 8 matches so that 2 squares are left.
   b. remove 8 matches so that 3 squares are left.
   c. remove 4 matches so that 5 squares of equal size are left.



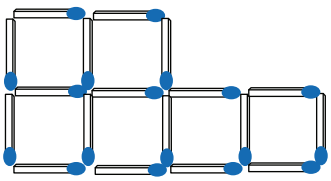
7. Remove from this figure 6 matches so that only 4 squares are left.



8. Remove 4 matches from these 6 squares so that 3 squares emerge.



9. Remove 6 matches from this figure so that 2 squares are left.

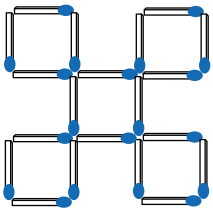


Name: Date:

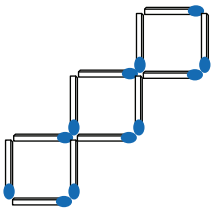
WS 54

Toss-up Tasks 2

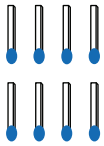
1. Shift 3 squares so that 7 squares of equal size emerge.



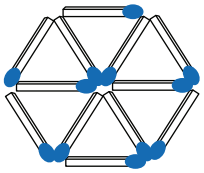
2. Shift 4 matches so that you get 4 squares of equal size.



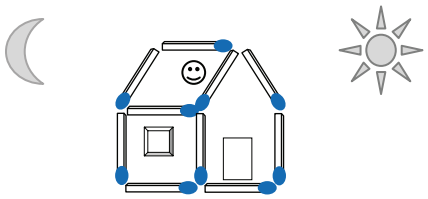
3. Shift 4 matches so that a cross emerges.



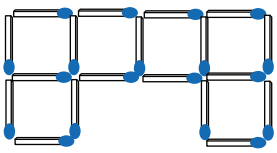
4. Shift 4 matches so that 3 triangles emerge.



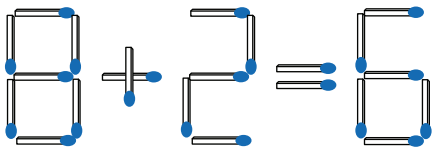
5. Elsa lives in the attic room. She can see the moon.
   Shift 2 matches so that she can see the sun.

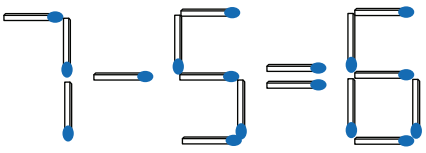


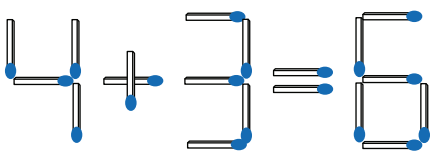
6. Shift 4 matches so that 5 squares emerge.

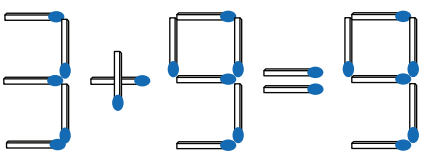


7. Shift 1 match in each calculation so that you get correct calculations.









11 Playing with Ruler and Compasses
Plane Figures

P. Ihn-Huber

Reference to Standards and Competences

Spaces and Shapes – Recognizing, naming and displaying geometric figures

- Making drawings with aids as well as free-hand drawing
- Making and analyzing models of plane figures (building, arranging, disarranging, putting together, cutting out, folding …)

Spaces and Shapes – Orienting in spaces:

- Recognizing, describing and using spatial relations

Patterns and Structures – Recognizing, describing and depicting regularities

- Recognizing, describing and continuing regularities in geometric patterns

Background and Didactic Commentary

Ruler and compasses constructions are highly stimulating and inspiring for children. When drawing with ruler and compasses the playful aspect should not be neglected, but at the same time accuracy has to be kept in mind. This accuracy is the difference between free-hand drawing and drawing with ruler and compasses, as precise circles, lines ... emerge.
Free-hand drawing prepares pupils for the use of the tools, as drawing using rules requires accurate working and trains the eye for spatial relations. The continuation of patterns and parquets demands complying with structures and rules.

But geometrical drawing also has artistic aspects. Especially contemporary artists often use geometric figures. The artwork of Marianne Hähnsen "Circles in Motion" could be used as inspiration. Here a ruler and compasses have been used, and the natural design from the colors provides the piece of art with vividness.
Dealing with the phenomena of optical illusions requires high accuracy in drawing and, in the creation process, provides insights into how optical illusions work. Surely the children will have to experiment a little until they reach a result that produces an optical illusion.

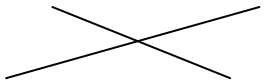
Methodological Advice

For WSs 55 and 56 ("Drawing with ruler and compasses"), it is suggested to provide children with white, plain sheets of paper. In addition to the suggested drawing tasks, the children will also find their own exercises. The main goal is to practice working with the drawing tools.
When dealing with the artwork "Circles in Motion" (WS 57) the class can discuss how the artist has divided the sheet into 9 rectangles of equal size. The circles or arcs are either completely colored or only until the next vertical or horizontal line. Circles with different centers don't overlap but are virtually layered.
For describing the patterns in WS 58, the children should be given help with the wording. To do so, the dots on the lines for the first pattern can be numbered 1-13 each in a clockwise direction. After this numbering dot 1 is connected with dot 1, dot 2 with dot 2, and so forth in the first pattern. In the second pattern, only the bottom dots should be connected. From the even-numbered dots arcs are drawn around the bottom right corner to the right edge. In the third, pattern arcs with a radius of the distance between 2 dots are drawn around the 3rd, 7th and 11th dots. For the children's own designs, we have provided empty squares to be used as worksheets on the CD-ROM.
Jointly reflecting on optical illusions as a class (WS 59) can be the motivation for drawing the pupils' own optical illusions. The children will need several attempts until the diagonals starting in task 1 start moving against each other. Similarly, the two vertical lines only shift when the intersecting lines meet exactly in the center. The two circles of equal size appear very differently when big and small circles are arranged all around the original circles. Certainly the children will find more optical illusions in appropriate books that they will copy with pleasure.

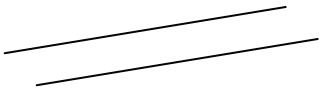
Name: Date:

Drawing with a Ruler

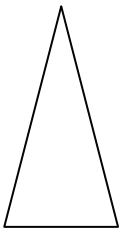
1. Draw with a ruler:
   lines that intersect,



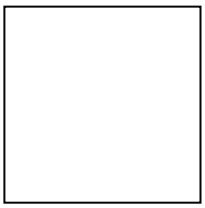
   lines that are parallel to each other.



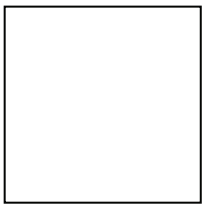
2. Draw different figures with your ruler.



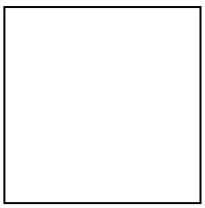
3. From this square, make …
   … 2 triangles with 1 line.



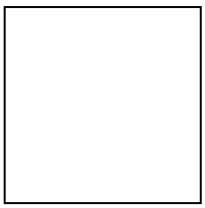
   … 3 triangles with 2 lines.



   … 1 triangle and 1 quadrilateral with 1 line.



   … 1 triangle and 1 pentagon with 1 line.



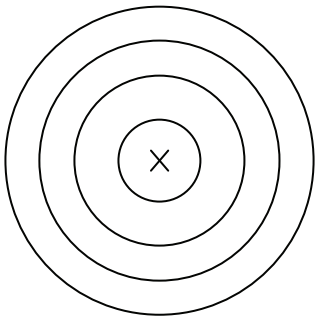
Name: Date:

WS 56

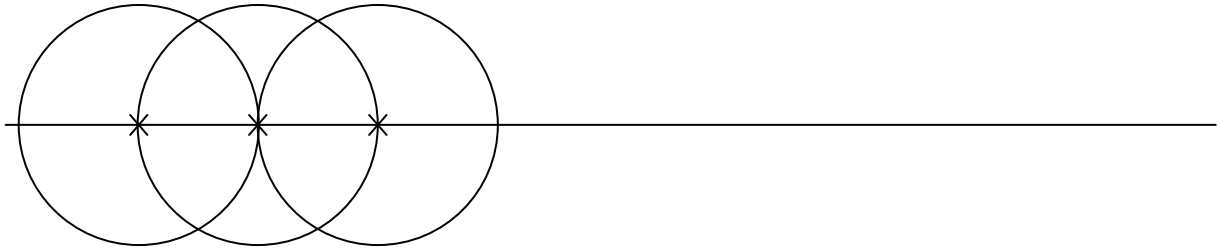
Drawing with Compasses

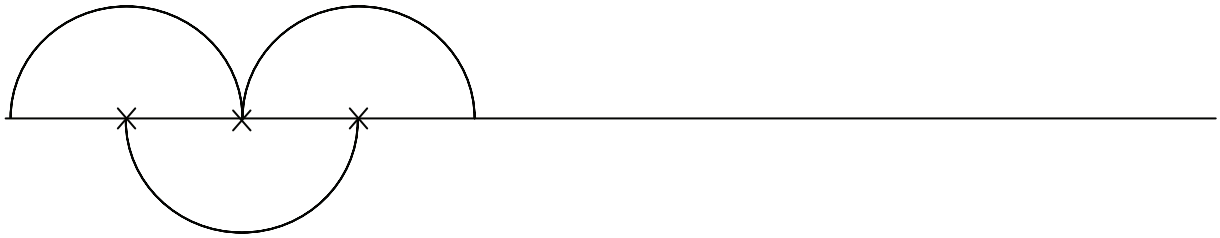
1. Draw different-sized circles.

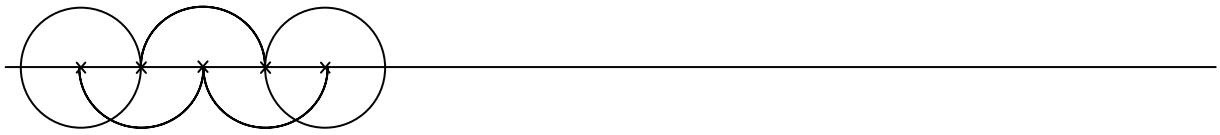
2. Circles that have the same centers are called “concentric circles“.
   Draw concentric circles here.



3. Extend the patterns.



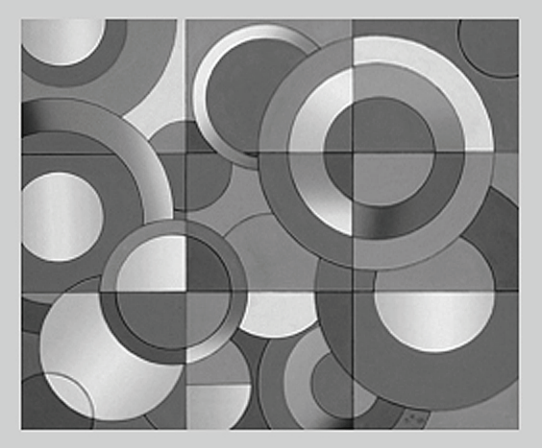




Name: Date:

“Circles in Motion“

1. Look at the picture.
   Try to describe how the artist Marianne Hähnsen has used compasses and a ruler.

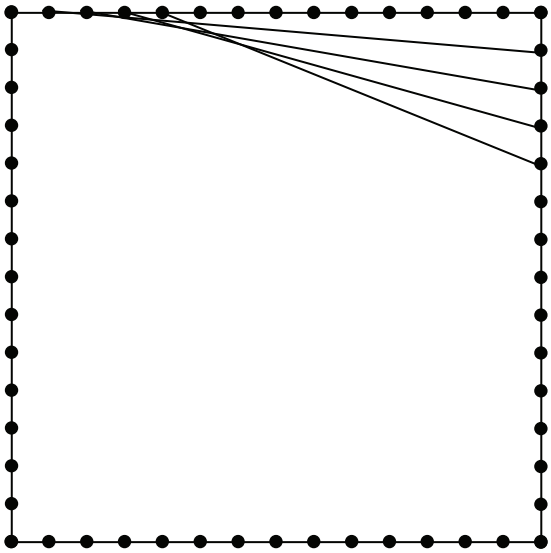


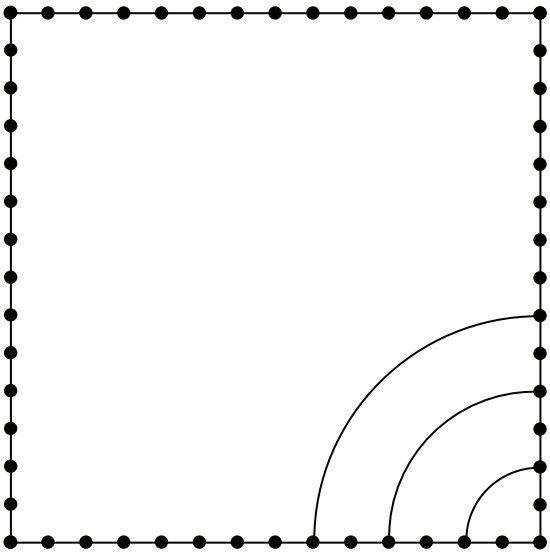
2. Now try yourself to draw a picture like Marianne Hähnsen.
   Use compasses, a ruler and bright colors.

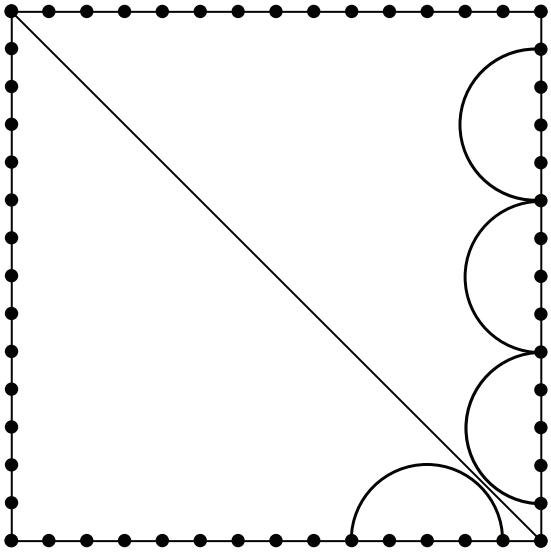
Name: Date:

Copy Patterns and Invent your Own

1. Look at the patterns and extend them. Describe what you have drawn. Color your patterns if you like.





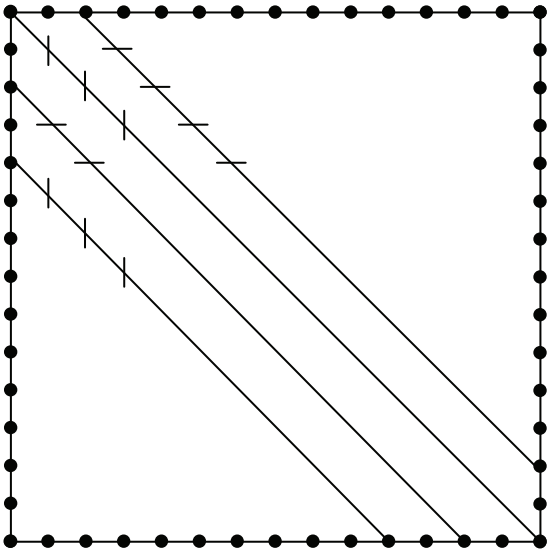


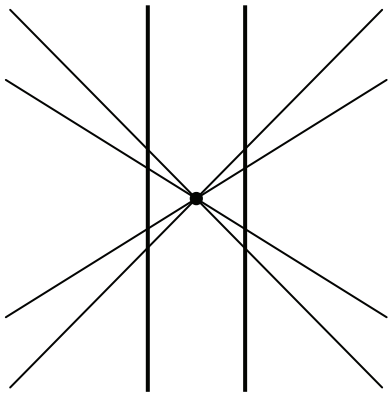
Name: Date:

Drawing Optical Illusions

1. First finish drawing the diagonals starting from every second dot from top left to bottom right. Then draw in the small lines – vertically on every second diagonal, horizontally on the others.

   Watch what is happening with the diagonals.

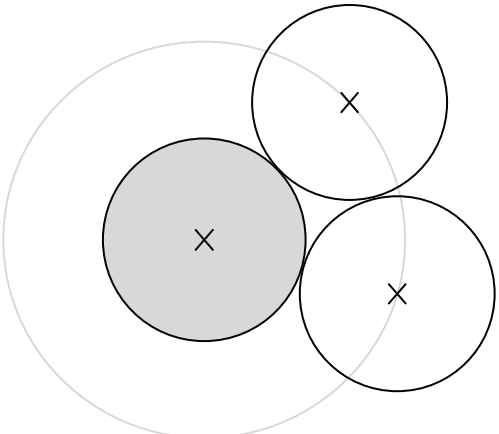


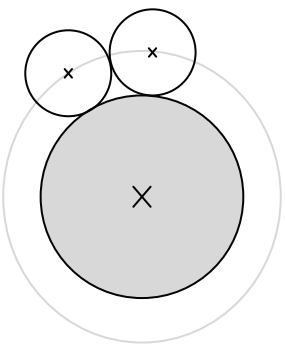


2. Draw many straight lines through the dot.

   Watch what happens with the vertical lines.

3. Draw circles, all of which touch the edge of the shaded center circles. Which center circle is bigger? Measure with the ruler.





12 Patterns and Ribbon Ornaments
Symmetries

H. Gutmann

Reference to Standards and Competences

Patterns and Structures – Recognizing, describing and depicting regularities:

- Recognizing, describing and perpetuating geometrical and arithmetical regularities
- Inventing, changing systematically and describing arithmetic and geometric patterns

Background and Didactic Commentary

Even young children come across patterns everywhere: in ever-recurring choruses in children's songs, when playing with building blocks, in nursery rhymes, in picture books. They are fascinated with these patterns and request repetition of what they have heard or being read again and again. They obviously enjoy the patterns in stories, the predictable, the included structure. Already at a preschool age, children love to draw patterns. They delight in recurring forms and colors. Children regard patterns mainly from an aesthetic point of view and are often fascinated by the beautiful results. Where does this fascination come from? Thinking itself is a structured activity. A structure offers security and at the same time serves as the base for key skills like creativity, communication and problem-solving. Mathematics is full of structures and patterns. It has been referred to as the science of patterns. By exploring patterns children practice recognizing the connections between positions, symmetries, identical forms and recurring regularities. Emotional perceptions of patterns are based on these geometrical features.

Now what exactly are patterns and ribbon ornaments? For both of them a basic element is arranged repeatedly by a certain rule. The repetition can be pictured geometrically by a mapping of congruencies. While patterns are units complete by themselves (e.g. carpet, clothes …), ribbon ornaments repeat themselves in one direction, theoretically infinitely (e.g. decorations on plates, braids …). A ribbon ornament is defined by a figure limited by two parallel straight lines with a repeating displacement.

Methodological Advice

First the children should act with different materials (geometrical small plates, arithmetical plates) and work with different patterns. Here the disambiguation arises by itself: What is a pattern? It is not so easy to put this into words, particularly when children are younger. Nevertheless the children, even in the 1st grade, manage to describe the characteristics of a pattern using simple words. "It comes again and again …", "It repeats itself …", "It's like a mirror." etc. Putting one's own thoughts and findings into words is important for the acquisition of the facts and is an educational principle.

After discovering the regularities, children should extend the patterns and create patterns themselves. They pay attention to a regular layout of forms and colors by their own accord. A square or triangle grid lends itself as an aid in drawing.

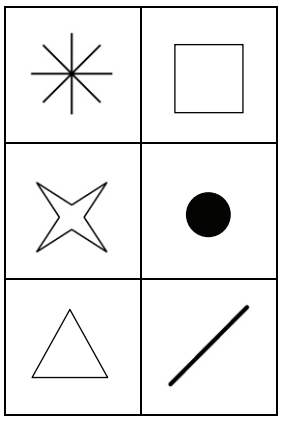
Educational contents consist of the recognizing, coloring, extending, describing, copying and inventing of patterns.

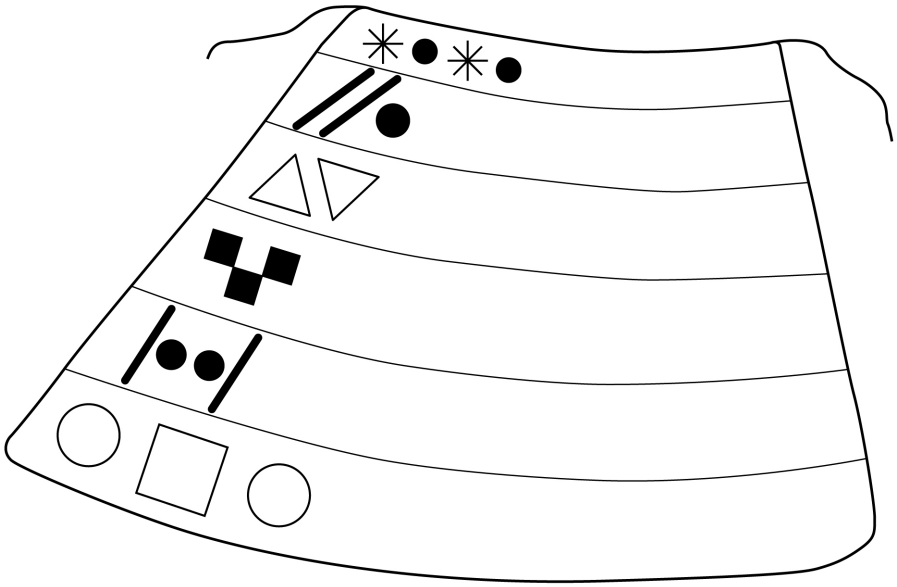
Name: Date:

The New Objects of Wizard Kuxbarux 1

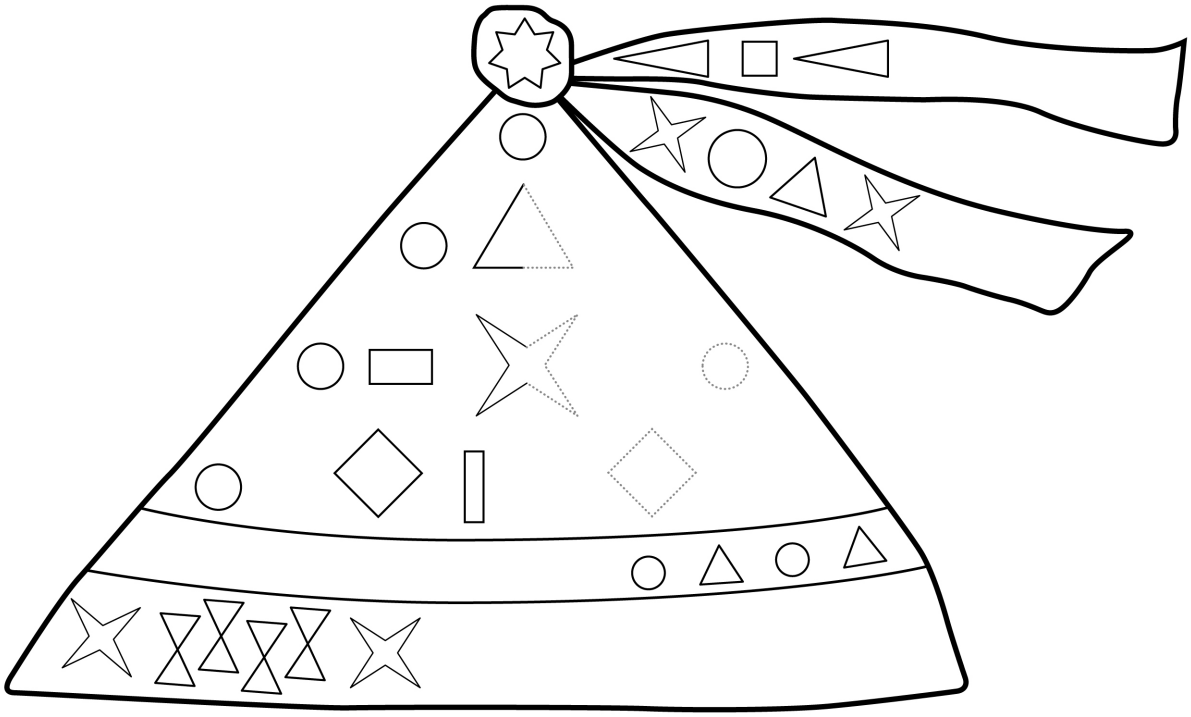
The wizard Kuxbarux is sitting on his old magic carpet thinking. Nobody remembers him anymore. He absolutely wants to be known to all children again. Therefore he needs new colorful magical objects!

1. First he needs a new charm cape. Continue to draw the patterns.
   Which of the following forms occur in the patterns of the charm cape? Tick them.





2. Also he must improve his magical hat!
   Extend the pattern and paint equal forms in equal colors.

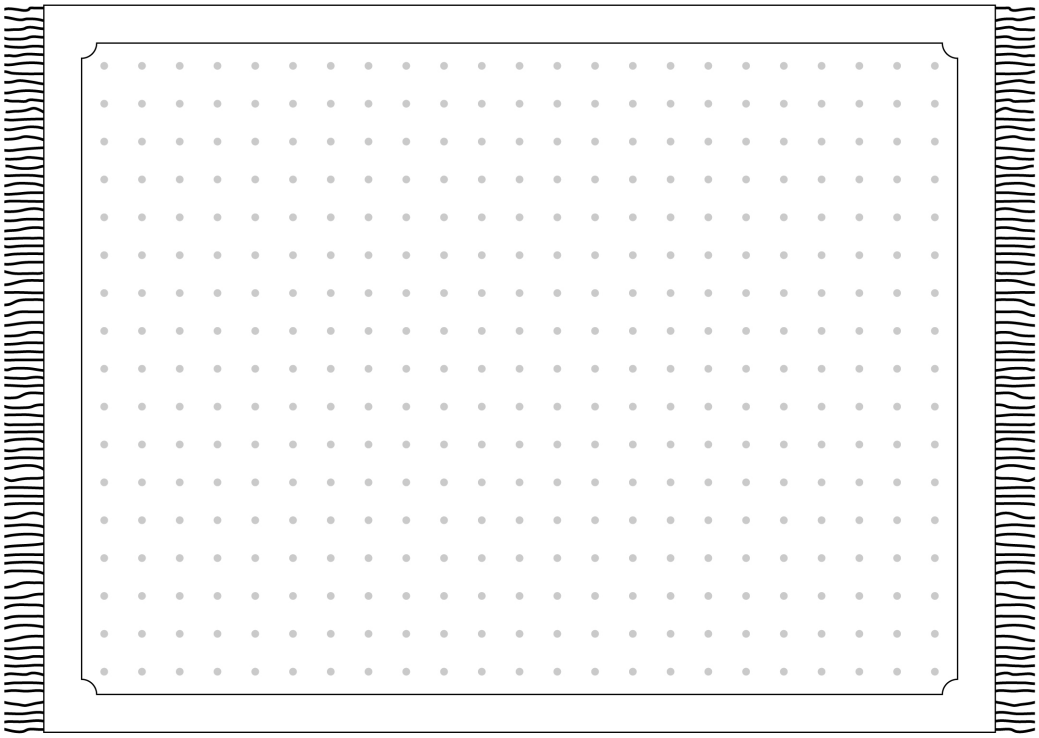


Name: Date:

WS 61

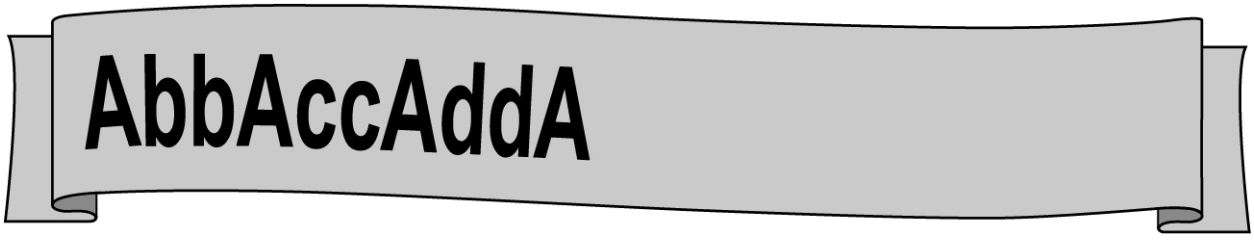
The New Objects of Wizard Kuxbarux 2

1. For his new magic carpet he doesn't have any more ideas. Help him and invent a pattern yourself. Use 3 different forms and 4 different colors.



Describe your pattern. What does it look like?

2. On his magic scarf the following is written:
How does it go on? Continue with the letters.



Describe the pattern.

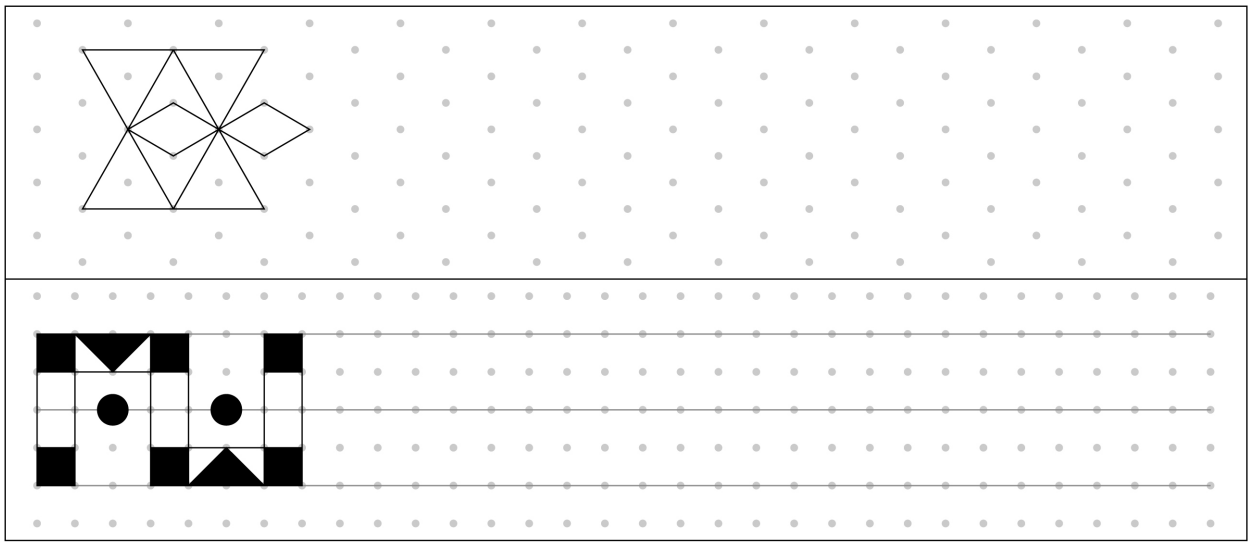
Name: Date:

WS 62

The Wizard's Party

Wizard Kuxbarux would like to have a big party for children. For this he still has a lot to prepare.

1. Wizard Kuxbarux wants to hang up two beautiful garlands.
   Unfortunately a part is missing. Finish drawing them and color them in.



What do you notice about the garlands?

2. At the entrance a welcome-sign is missing. Finish drawing it.

Name: Date:

Wizard Kuxbarux and the Pattern Game 1

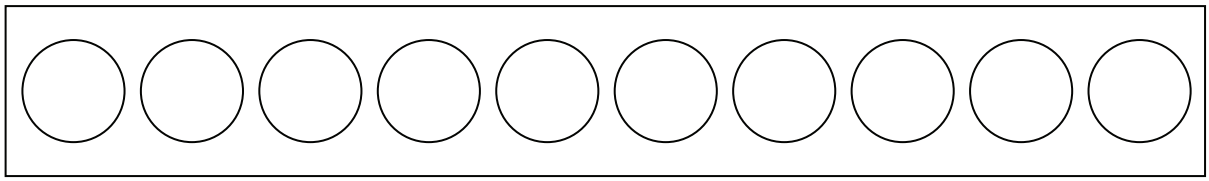
The wizard Kuxbarux loves to play “inventing patterns“. For this he needs red and blue little plates. He will explain to you how it works.

1. Copy the pattern with little plates.
   (R is a red plate, B a blue plate).

R	R	B	R	R	B	R	R	B

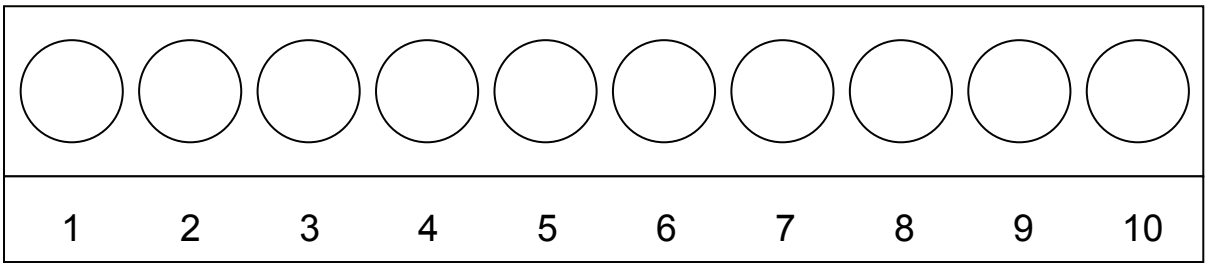
Explain accurately what the pattern looks like.

2. Place a pattern using plates on the table yourself. Sketch it in colors on the template.



Your partner should now copy the pattern and extend it by 10 plates.

3. Each plate sits at a certain point of a numerical series.



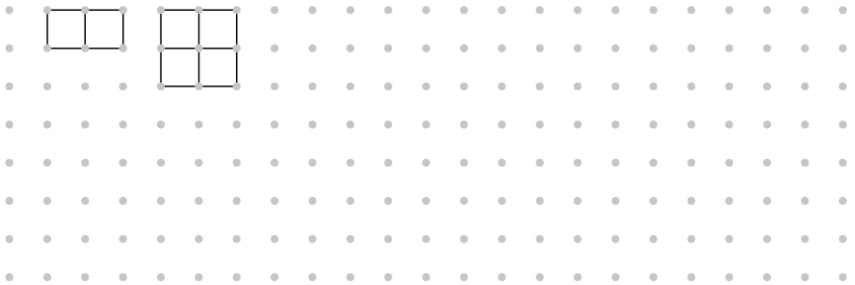
Create a pattern using the plates and sketch it in color into the template.
At which point does the color always change?
Write down the numbers of those points.

What do you notice about this numerical series?

Name: Date:

Wizard Kuxbarux and the Pattern Game 2

1. The wizard continues by saying:
   Here you can see the beginning of my new pattern.



   How does it continue? Sketch it!

2. How many boxes are there in each figure? Write down the numbers as a numerical series.

   What do you notice?

3. Invent a pattern using squares only.

   Explain your pattern.

   Can you also describe your pattern with numerals?

13 Line Symmetry – Radial Symmetry
Symmetries

E. Kick

Reference to Standards and Competences

Spaces and Shapes:
- Recognizing, describing and using line symmetry
- Extending and creating symmetrical patterns

Patterns and Structures:
- Creating geometric patterns, altering them systematically and describing them

Background and Didactic Commentary

By the time they enter school, children have experienced a lot in the field of symmetry – consciously or unconsciously. This includes children looking in the mirror facing their own reflection as well as the recognition of a vertical body axis. The environment also shows symmetric figures (leaves, blossoms, insects, towers, bridges …) in abundance.
Moreover, symmetrical forms are often perceived as being very clearly structured, beautiful and aesthetic. Creative artists throughout the ages have captured and transformed the conception of symmetry in their rosettes, ornaments and mandalas.

It seems significant that our brain can recognize and store structures of symmetrical forms faster than those of asymmetrical forms. In the arithmetic field, this is seen extensively when symmetrically displayed and structured numeral pictures are compared to an unsorted amount of dots.
In primary school mathematics, a figure is called symmetrical when it comes to congruence with itself by:
- reflection over a mirror axis (axis of symmetry),
- a rotation (quarter turn, half turn, three-quarter turn, etc.) about a point, or
- shifting by a prescribed length into a certain direction.

Methodological Advice

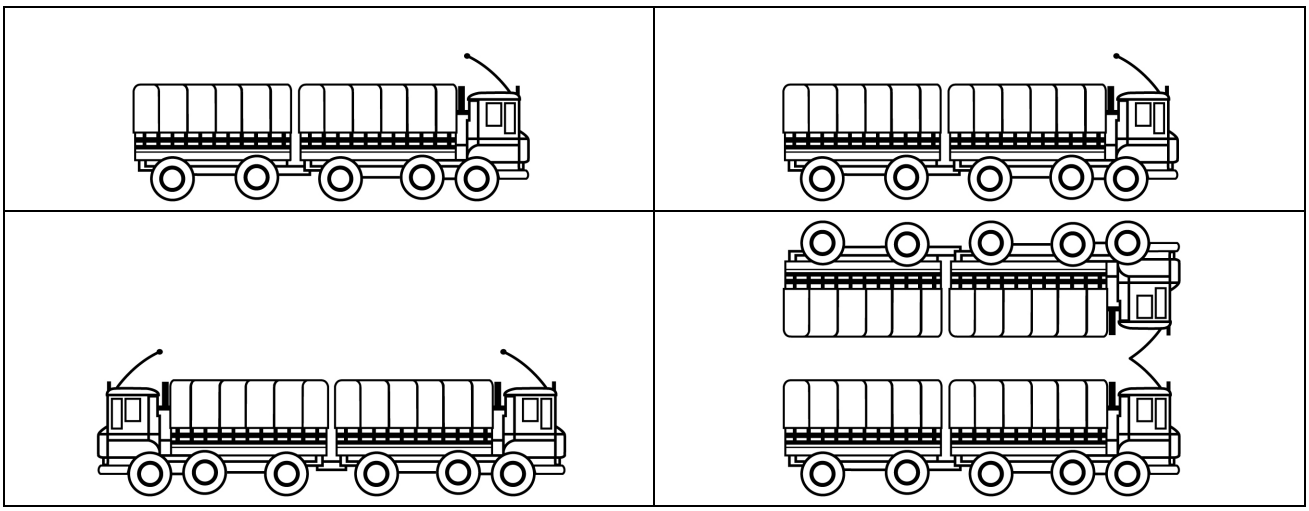
Even preschool-aged children gain insights into symmetry through various activities. This experience is now picked up and expanded further. Fundamentally significant in this section are techniques using folding and cutting. Further popular techniques to make axis-symmetrical figures are blobbed pictures or figures sketched onto a folded sheet where dots are being punctured with a needle.

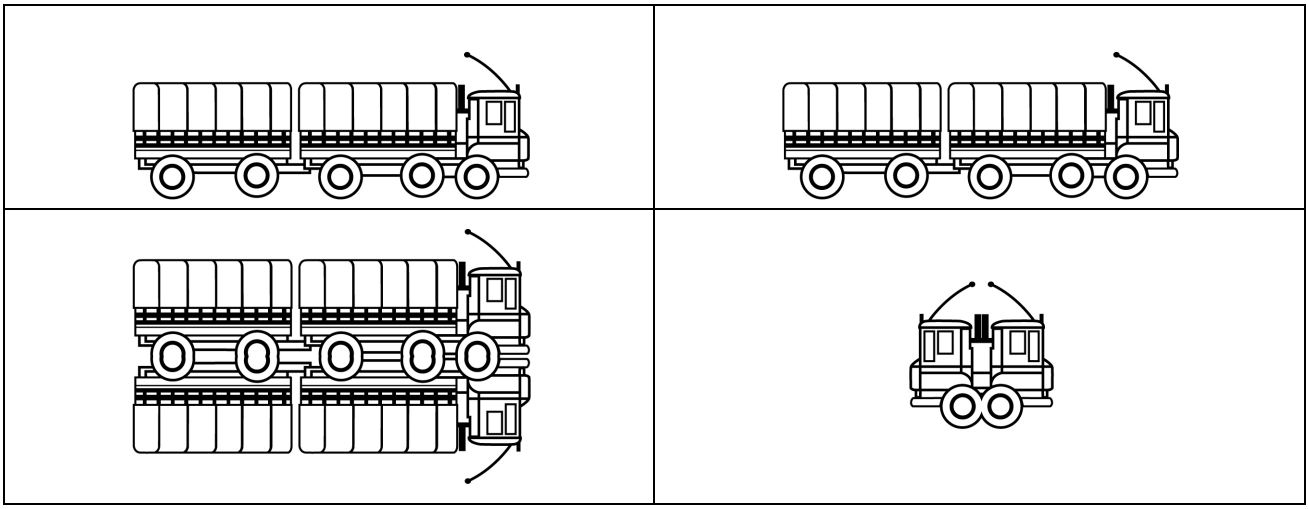
Afterwards, children can discover quaint and funny pictures using WS 65 and playing with a mirror. To do so, the children put one edge of their mirror vertically onto the picture and find the displayed target figure by rotating and shifting. WS 66 takes into account that vertical, horizontal and diagonal mirror axes exist. Furthermore, figures with multiple axes of symmetry are imaginable. The pupils use all the knowledge they have gained so far when reflecting using the geometric board (WS 67) and try to translate their solutions into drawings. In WS 68 the children complete objective figures and abstract patterns on squared paper. Exercises which are done with a partner as well as reflecting over various axes top off this chapter on line symmetry. WS 69 brings the eye to radial symmetry.

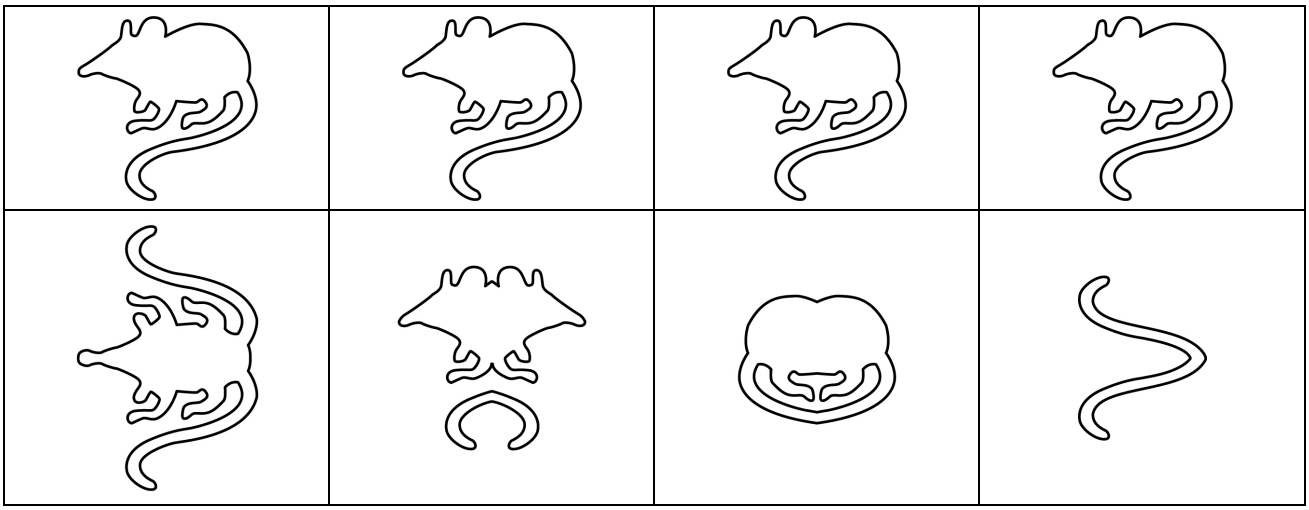
Name: Date:

Special Reflections

Where do you have to place the mirror on the truck or the mouse in order to obtain the figure below? Draw it in.



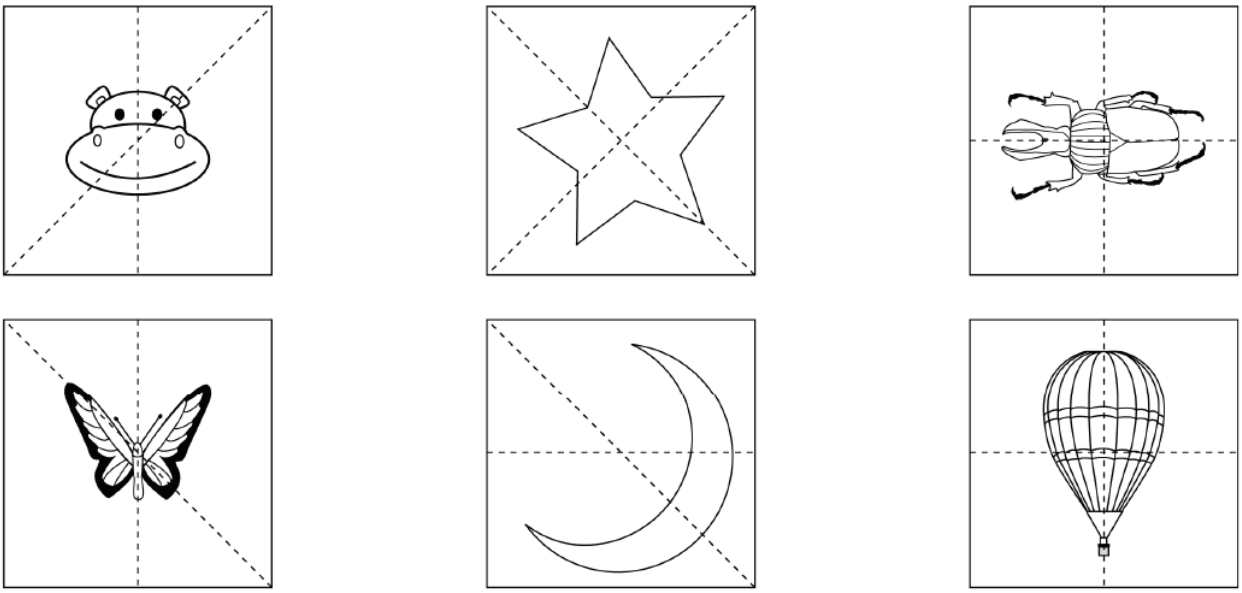




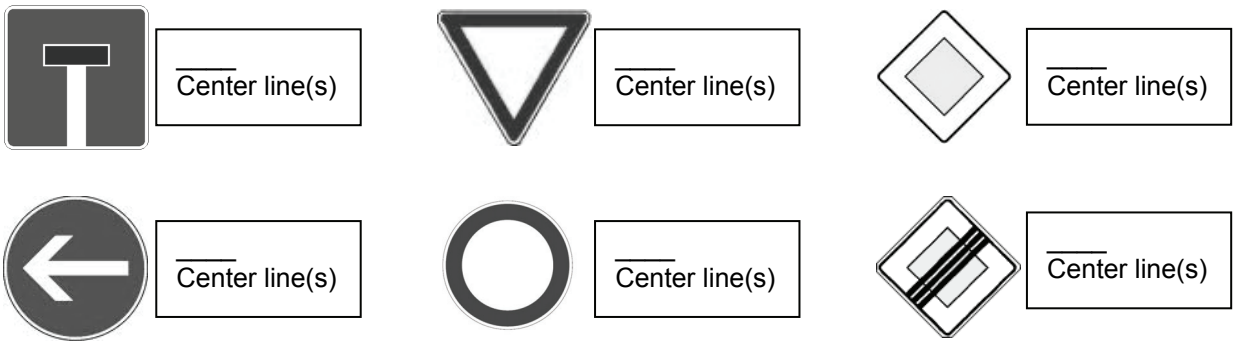
Name: Date:

Drawing Center Lines

1. Which center line is correct? Trace it with a colored pencil. (Check with a mirror.)



2. Check the following traffic signs for their center lines and draw them in. Write down the number.



3. Letters and words can also have a center line. Draw in the axes you find.

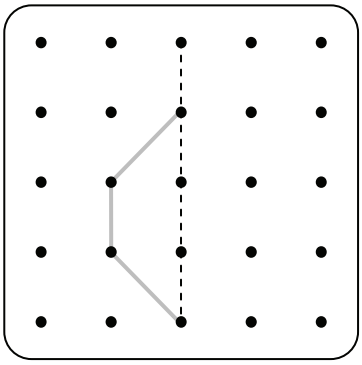
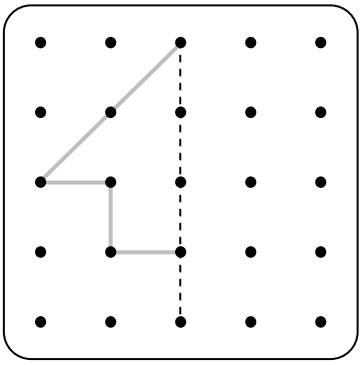
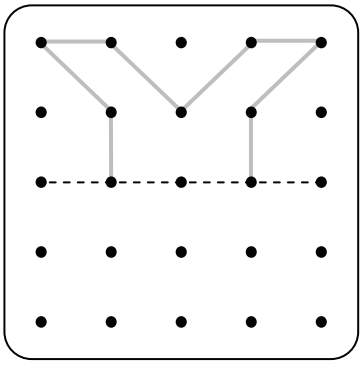
A C W P E M

BOB ANNA MUM

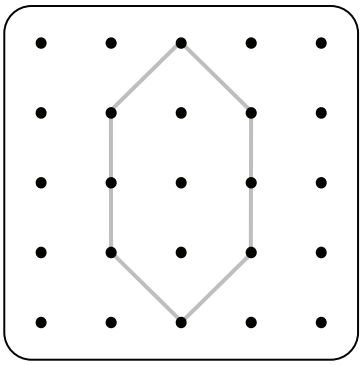
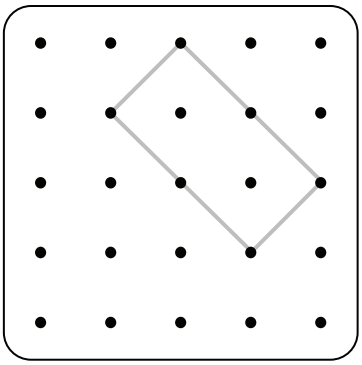
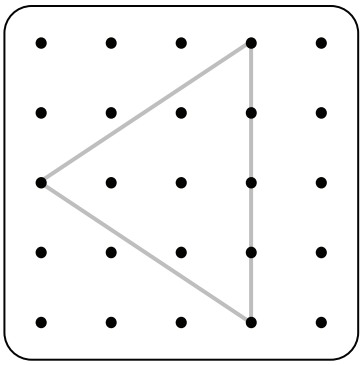
Name: Date:

Reflecting Planes

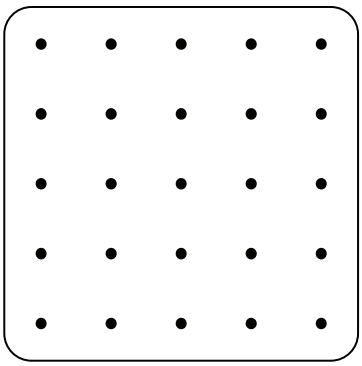
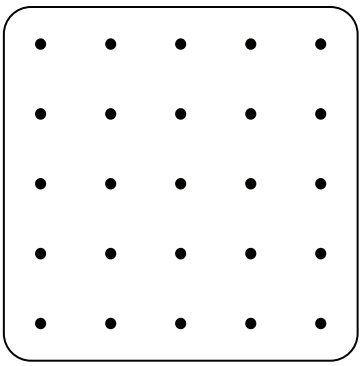
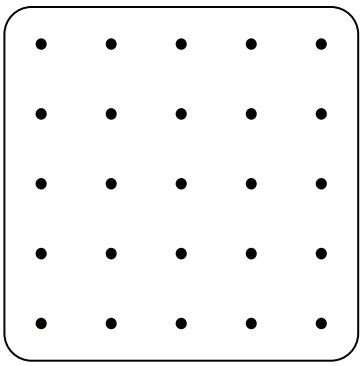
1. First create the center line on your geometrical board (dashed line). Then move the rubber band to create the given figure. Reflect it on your geometric board. Draw it in.

2. Define the mirror axis/axes of each figure and draw it in. You can check with a mirror.

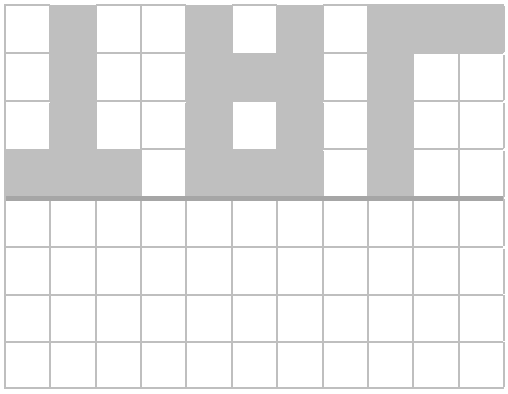
3. Invent figures that have a mirror axis and sketch the figures with their axes.

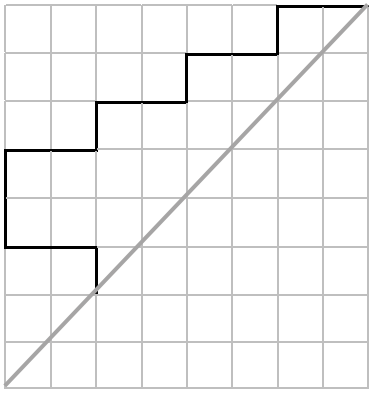




Name: Date:

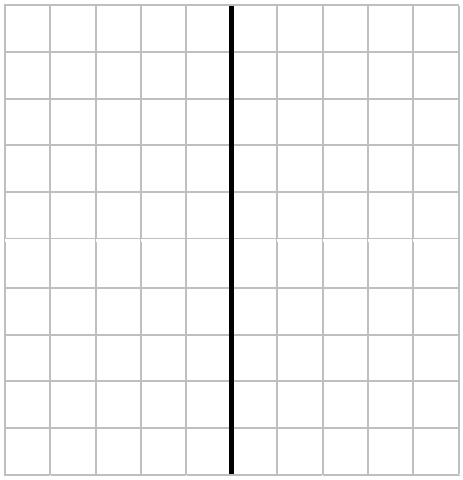
Reflecting Over One or More Axes

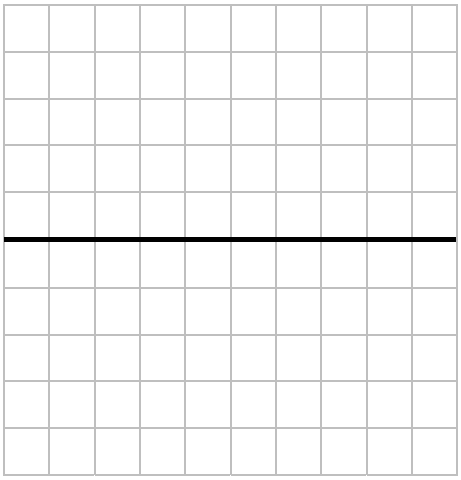
1. Complete the missing half.



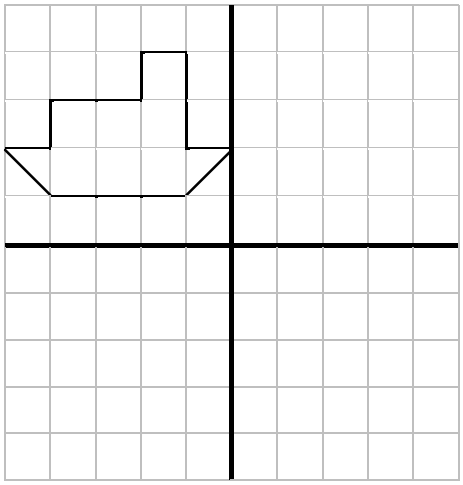


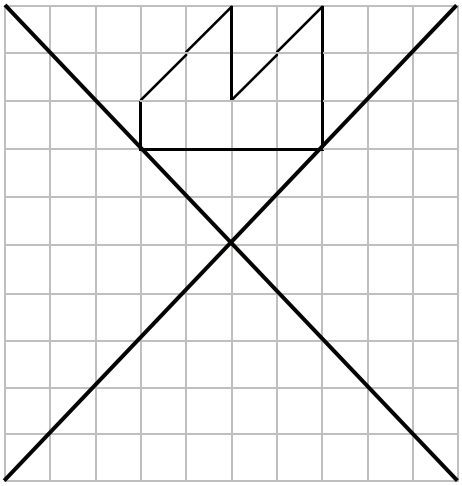
2. Reflect over the mirror axis the pattern that your partner gives you.





3. Reflect the given pattern over both mirror axes.

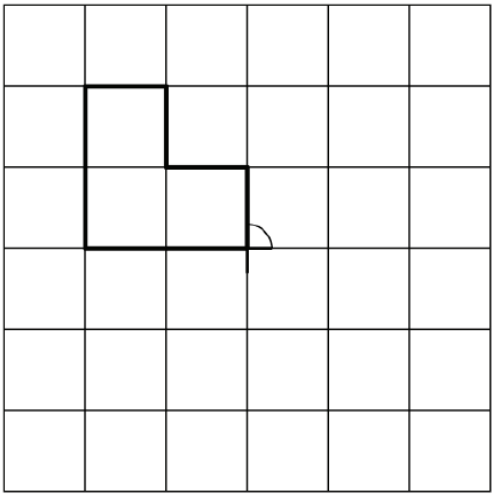


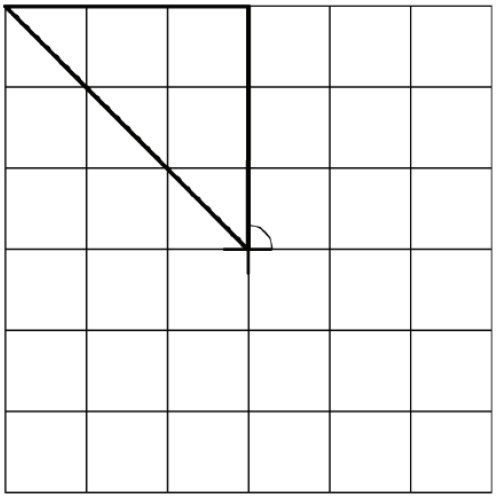


Name: Date:

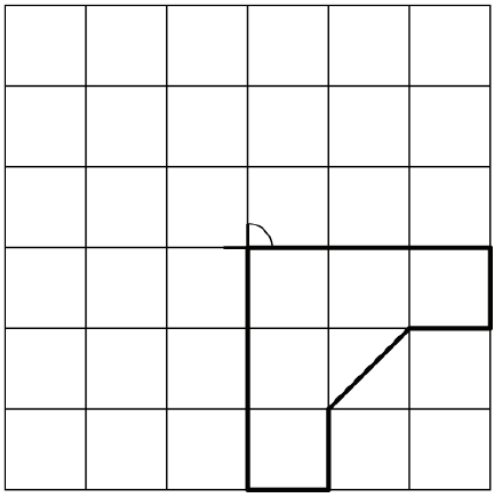
Radial Symmetric Figures

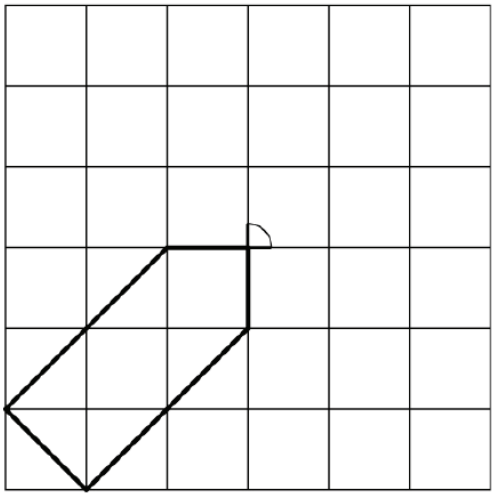
1. Rotate the following figures repeatedly by a quarter turn.



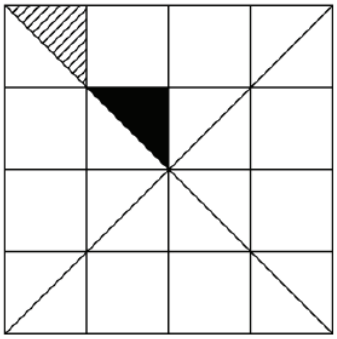


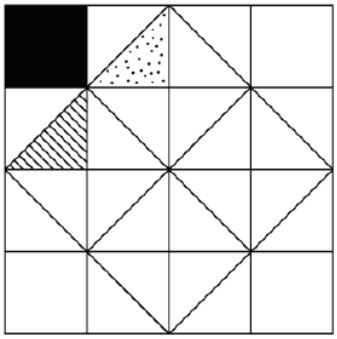
2. Rotate the figures. Where do they stop after a quarter turn, a half turn and after a three-quarter turn?

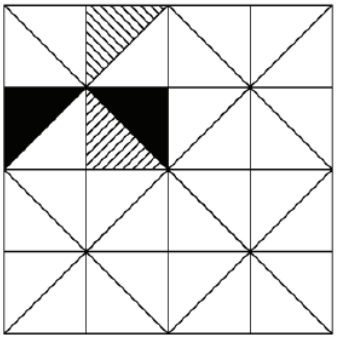




3. How do you have to color the squares to receive a rotationally symmetric figure?







14 Parquets/Tiling
Symmetries

R. Dolenc-Petz

Reference to Standards and Competences

Spaces and Shapes:

- Recognizing, naming and depicting geometric forms

Patterns and Structures:

- Recognizing, naming and depicting regularities in geometric patterns
- Inventing own geometric patterns as well as changing them systematically and describing them

Background and Didactic Commentary

Parquets are created by covering a plane with congruent figures. In doing so, no omissions or overlapping may occur. Parquets are characterized by the repetition of a chosen motif in all directions. Here the starting figure can be shifted, turned or reflected. One can distinguish between simple parquets (with one congruent figure) or parquets with two or more congruent figures.
Parquets are found frequently in the environment (checkerboard, honeycomb, brick floorings …).

In addition to supporting aesthetics and creativity, both of which make children love tiling, the examination of parquets trains pupils in spatial awareness. Important basic knowledge concerning special features of regular and irregular figures, congruency, shifting, rotation and reflection are extended.

Methodological Advice

Parquets from simple figures (squares, triangles) are particularly useful when introducing this topic, as working tangibly with paper squares cut into triangles and rectangles is possible. Patterns can be created and then transferred and continued on squared paper. This activity challenges children to find out which figures cover a plane completely. Here stencils can also be used for the children to actively explore, in addition to the tasks addressed in this section.
WS 70 addresses drawing a parquet using one kind of figure (squares). The combination of two different figures or the tiling of more complex figures helps to extend upon the concept tiling.
The tasks in WS 71 are suitable to revise terminology related to planes and position by describing the emerged patterns in discussion.
Experience shows children begin orient themselves using the squares of gridded paper. Therefore they should often be inspired to invent their own patterns in addition to those presented in the worksheets. If the diagonals of the squares are included in the activities, additional and more demanding options emerge. The tasks in WS 72 have been created to inspire this. On the blank WS 74 children can invent their own patterns on triangular paper.

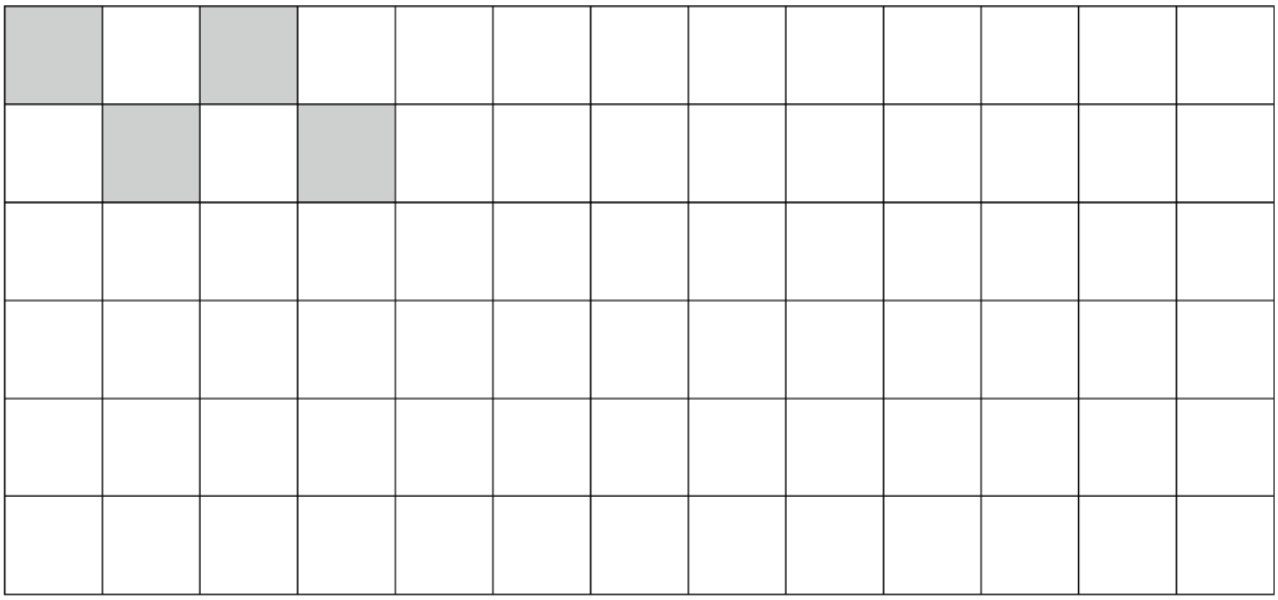
After the children have extended the patterns in the worksheets as well as invented their own, the tasks in WS 73 inspire the drawing of parquets with geometrical and concrete figures. Making parquets from concrete figures is a very demanding but at the same time a motivating activity. Figures have to be shifted and turned in the pupils' imagination and recorded on the sheet.

Name: Date:

WS 70

Parquets Using Squares

1. Extend this pattern using squares.

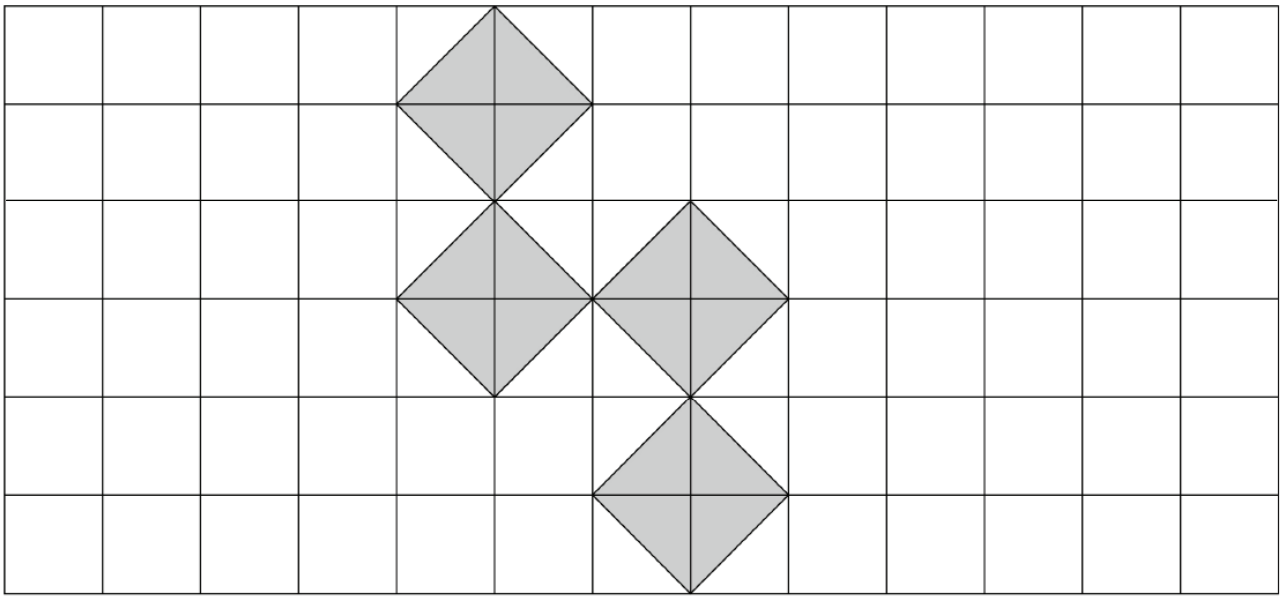


Where do you know this pattern from?

_____

_____

2. Extend this parquet in all directions.



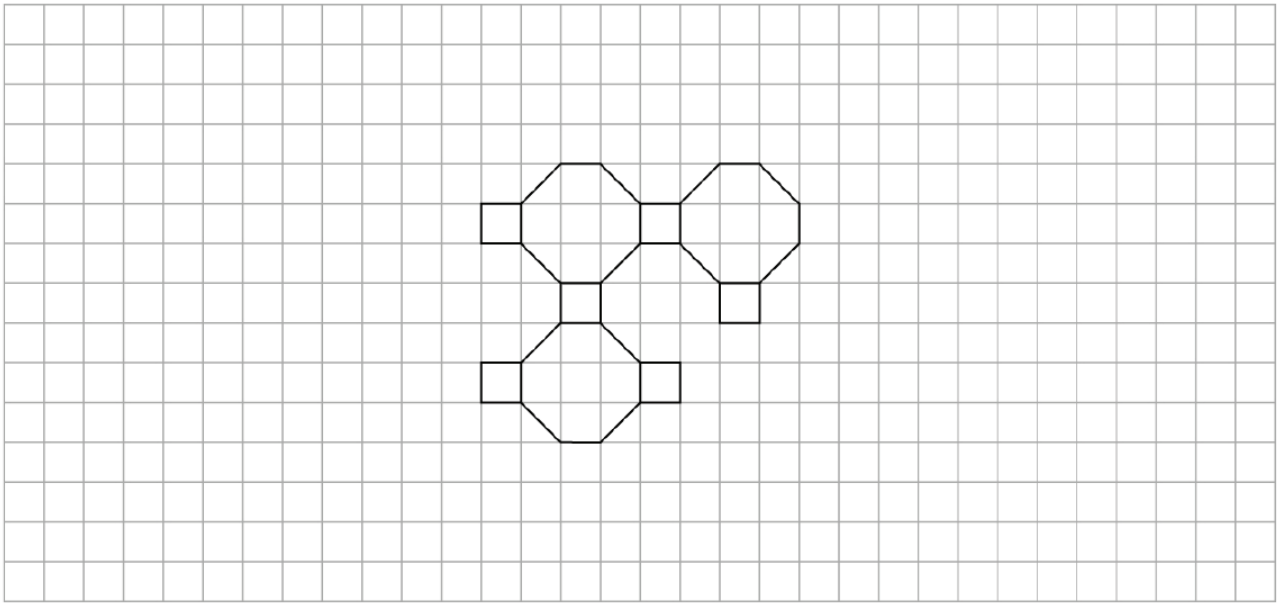
3. Invent your own patterns using squares and sketch them on squared paper.

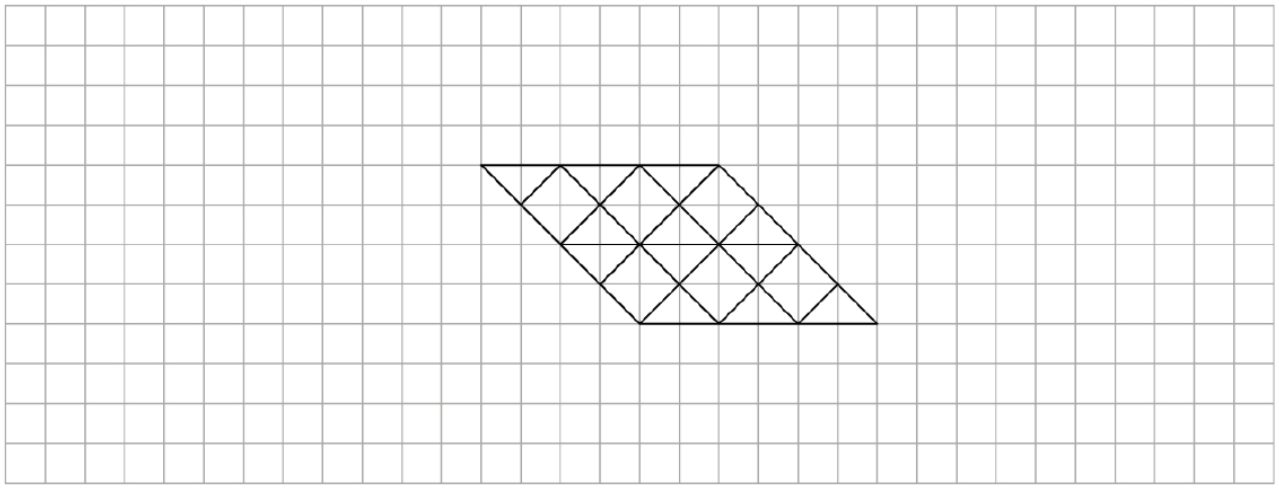
4. Start a parquet. Have a classmate extend it.

Name: Date:

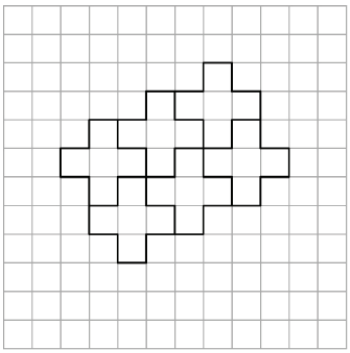
Proceed with Drawing Parquets 1

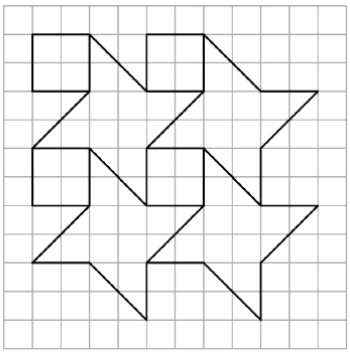
1. Extend the started parquets in all directions. Color them in.





2. Transfer the started parquets onto squared paper. Extend them.



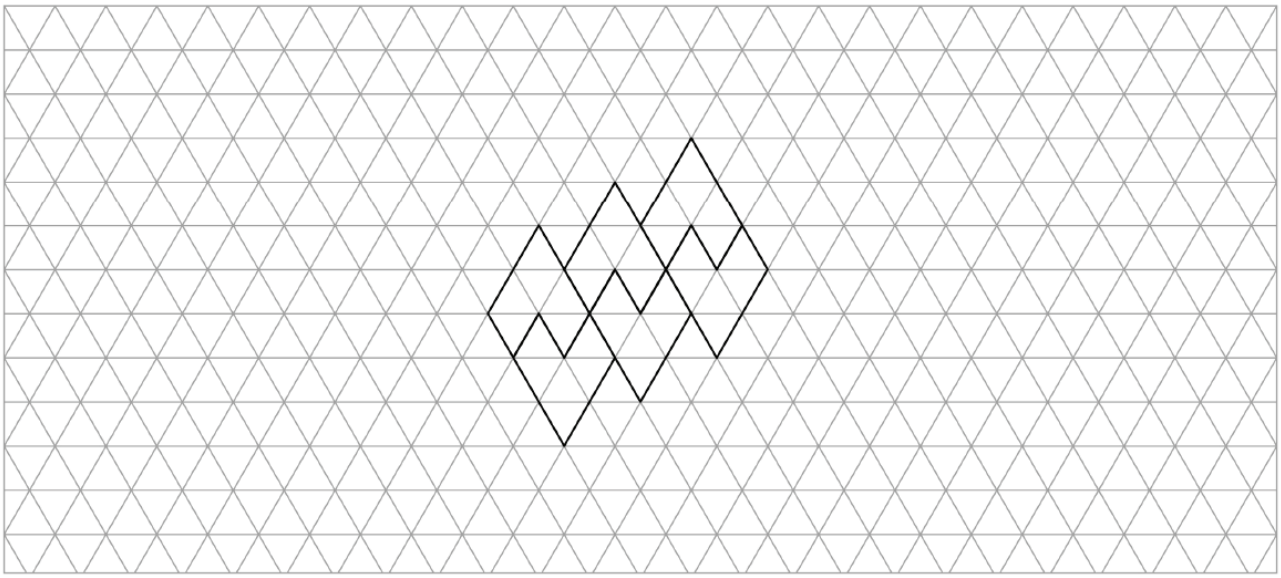


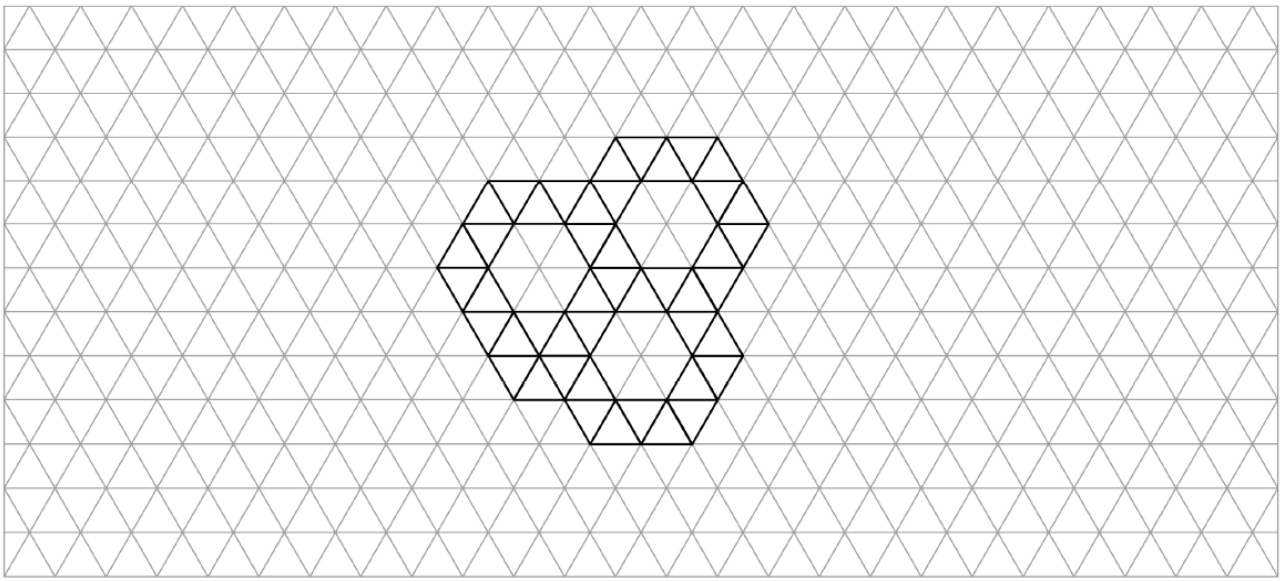
Name: Date:

WS 72

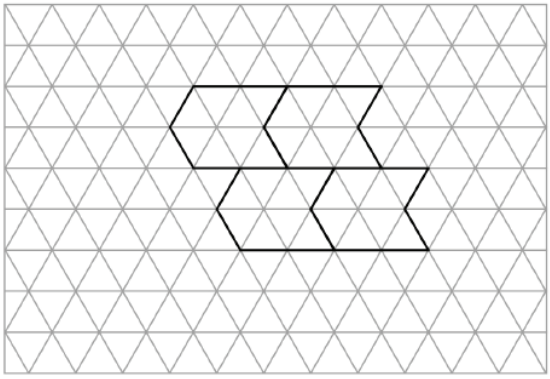
Proceed with Drawing Parquets 2

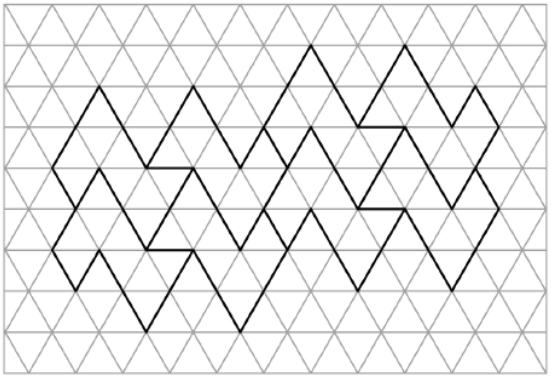
1. Extend the started parquets in all directions.





2. Transfer the started parquets onto triangular paper. Extend them.



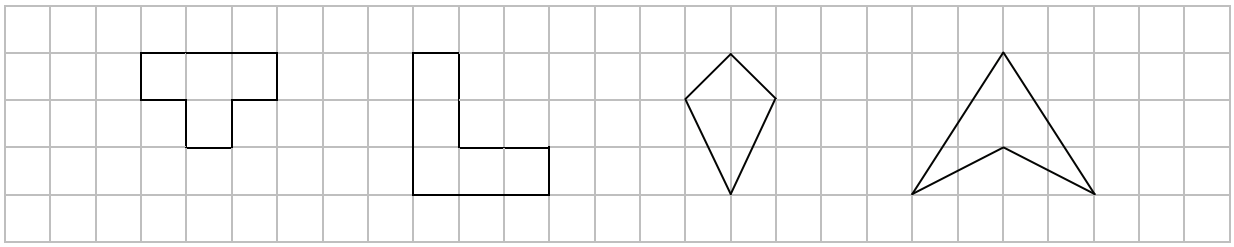


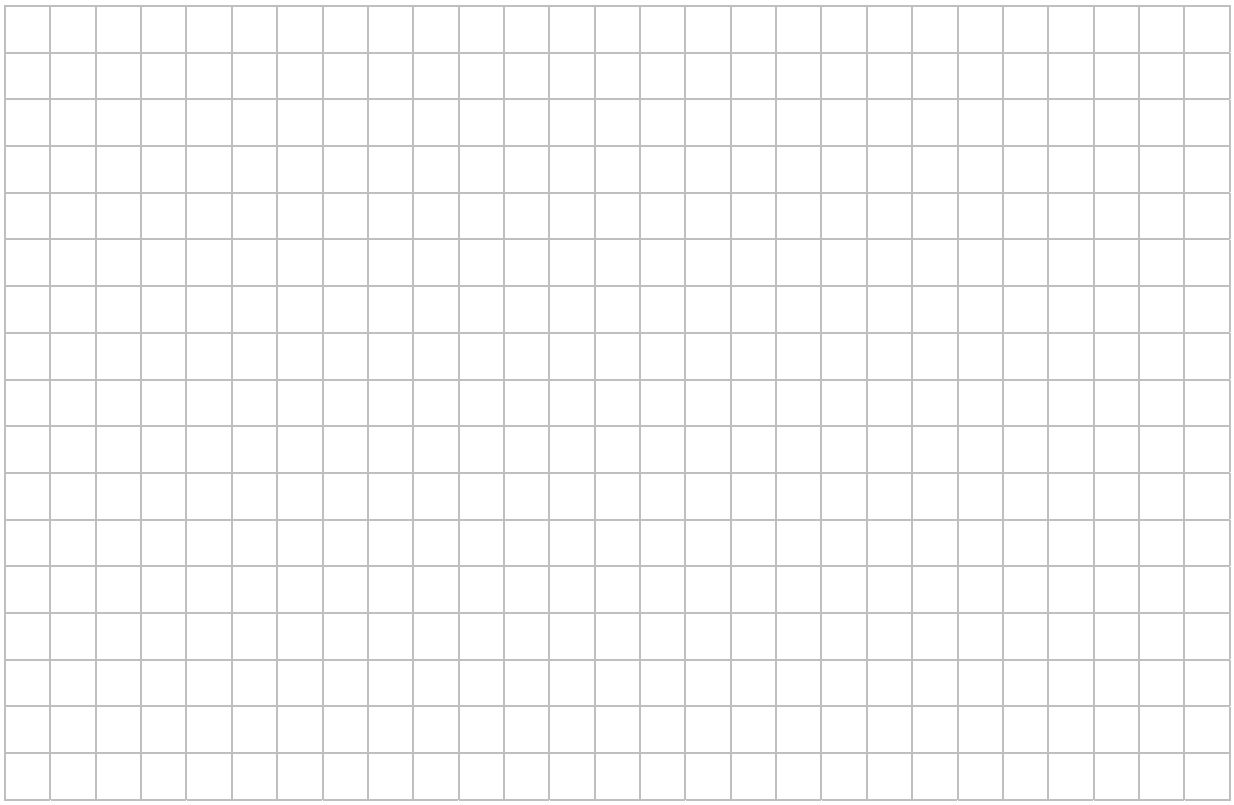
Name: Date:

WS 73

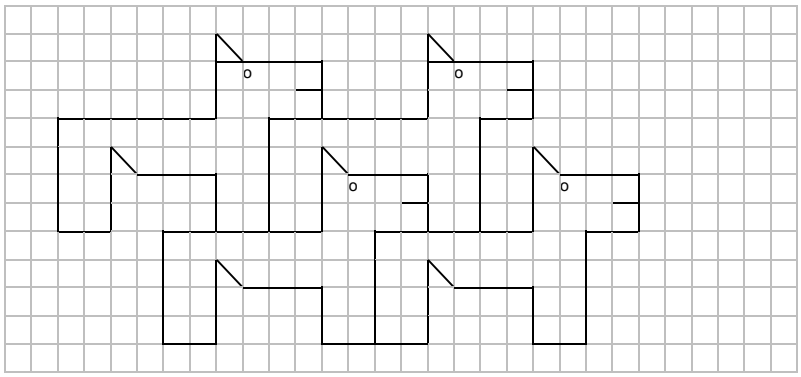
Drawing Parquets

1. Chose a starting figure. Sketch a parquet.
   Remember that you can turn, reflect or shift the figure.





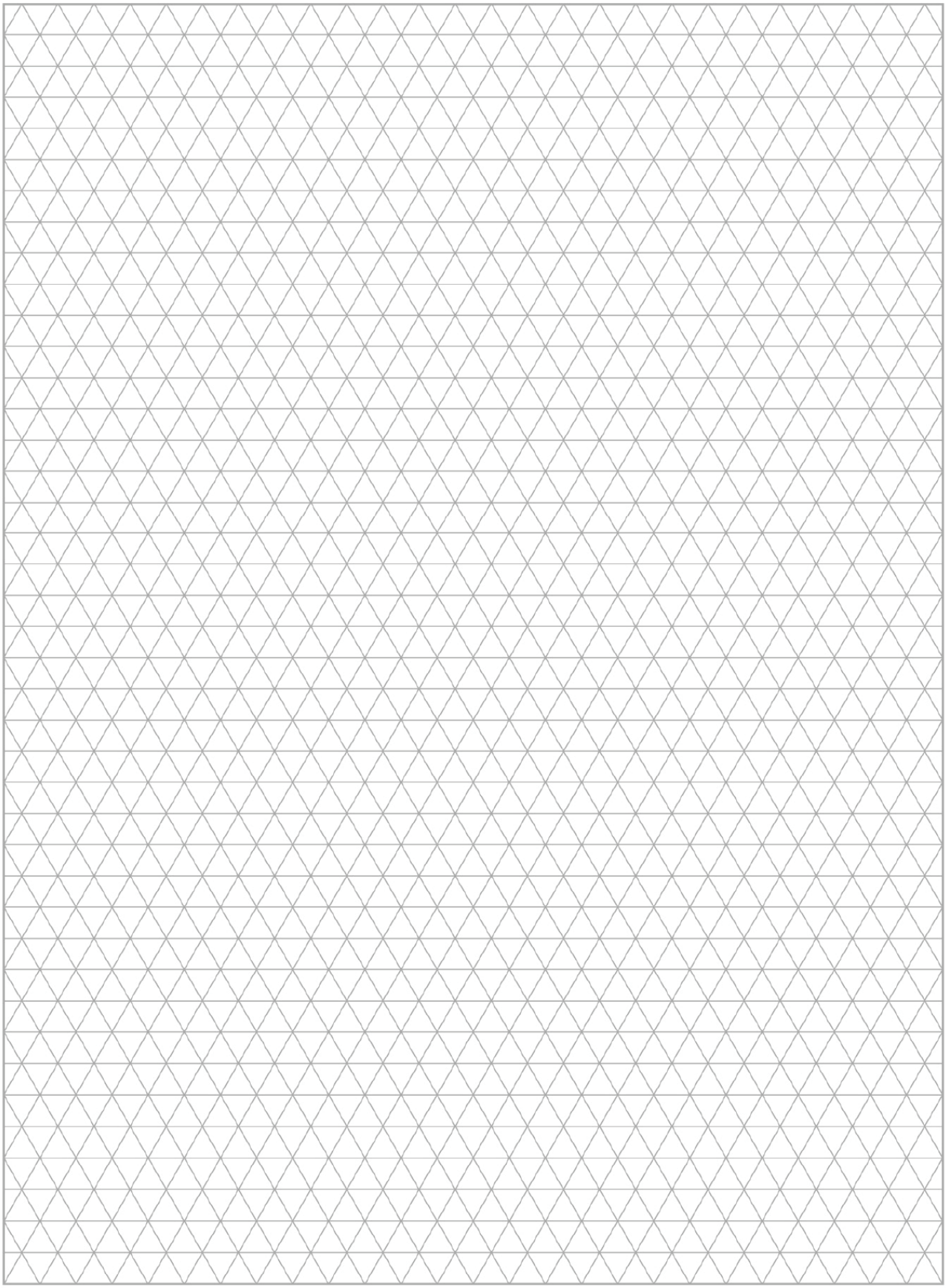
2. Transfer this animal parquet onto squared paper and extend it.



3. Invent your own parquet using animals, plants, houses, etc.

Name: Date:

Worksheet Triangular Paper



Further Reading

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