# Developmental Research in Inquiry-Based Mathematics Teaching 

Barbara Jaworski<br>Mathematics Education Centre

## Structure

- What does it mean to teach mathematics?
- Inquiry in the classroom
- The role of the teacher
- Inquiry in theory
- Developmental research
- New knowledge in theory and in practice


## Teaching

- What do I mean by teaching?
- What sort of teaching am I talking about?
- What are the aims of teaching?


## Teaching is ...

[not attending to this] is to accept a limited and impoverished understanding of teaching. (Pring, 2004, p. 22)
"An action might be described as 'teaching' if, first, it aims to bring about learning, second, it takes account of where the learner is at, and, third it has regard for the nature of what has to be learnt. (Pring, 2000, p. 23) -By a practice [e.g., teaching] I mean a collection of different activities that are united in some common purpose, embody certain values and make each of the component activities intelligible. (Pring, 2000, p.27)
-For example, the teaching of a particular concept in mathematics can be understood only within a broader picture of what it means to think mathematically, and its significance and value can be understood only within the wider evaluation of the mathematics programme. (Pring, 2000, p.27)
"[not attending to this] is to accept a limited and impoverished understanding of teaching. (Pring, 2004, p. 22)

## ... a moral practice

Teaching ... is more than a set of specific actions in which a particular person is helped to learn this or that. It is an activity in which the teacher is sharing in a moral enterprise, namely the initiation of young people into a worthwhile way of seeing the world ...there can be no avoidance ... of that essentially moral judgement of the teacher over what is worth learning and what are the worthwhile ways of pursuing it. (Pring, 2004, p. 18)

## Teaching for conceptual understanding

- The initiation of young people into a worthwhile way of seeing the world - the broader picture.
- The moral judgement of the teacher over what is worth learning and what are the worthwhile ways of pursuing it.
- What is worthwhile in mathematics?
- to do mathematics
- to use mathematics
- to apply mathematics


## Require mathematical understanding

## From mathematics to mathematical understanding

## Learning Mathematics

## Mathematics

Teaching Mathematics

Teacher has to transform mathematical knowledge into opportunities for the learner to learn mathematics with understanding (Didactics of Mathematics)

- How does/can the teacher do this?
- How does/can this knowledge/skill develop? University


# Bringing inquiry into the classroom 

 University
## Inquiry in the classroom

Wonder

## STUDENTS!

Explore

Imagine
Ask questions and seek answers

Investigate
Recognise problems and seek solutions

Reason

Discuss

## WHO?

## Inquiry-based tasks

- Provide easy access to mathematical ideas
- Enable everyone to make a start
- Provide opportunity to ask questions, solve problems, imagine, explore, investigate ...
- Encourage discussion and reasoning, diverse directions and levels of thinking, fluidity and flexibilility
- Encourage student centrality/ownership in/of the mathematics
- Promote serious mathematical thinking


# What kinds of tasks am I talking about? 

## Some examples ...

## Some inquiry-based tasks

A. Take any natural number and express it as a sum in as many ways as possible (e.g. $10=2+8=3+2+2+2+1=$ $1+1+1+1+6$ )
Multiply the numbers of any sum and find the largest product. Generalise?
B. You have a number of balls in a bag, some red and some white. What is the least number of red balls for which the probability of drawing two red balls in succession (without replacement) is more than $1 / 3$ ? Generalise?
C. Can you find any plane shape for which the area is numerically equal to its perimeter? Generalise?

Fibonacci Conference April 2012 -- Inquiry in the classroom University

## How tall a mirror must you buy if you want to be able to see your full vertical image?

## Grade 1

One student faces a mirror holding a stick (against his stomach). This student directs another, who, using a whiteboard marker, marks the mirror image the first one sees. Compare the original stick with the marks on the mirror. Try different distances from the mirror.

## Grade 2

One student holds a geometric figure (against the stomach) and explains to another student how to draw (on the mirror) the mirror image he sees. Compare.

## Grade 3

Measure yourself in centimetres. Measure your mirror image in centimetres.

Draw yourself seeing yourself in a mirror.

Grade 4
Have a mirror with a grid. One student holds a geometrical figure (against
stomach) and explains how another student can draw this on the mirror. Count number of squares (area) and compare.
(Jaworski, Goodchild, Eriksen \& Daland, 2011)

## Grade 11

## Grade 6

Measure yourself and your mirror image.Draw yourself (simplified) looking ina mirror with the correct ratios (and angles) in your drawing.

## Grade 7

Draw model of a figure and an eye and the mirror image the eye sees (keep the
eye and the figure at the same distance from the mirror?). Describe lengths and angles. What do you see?

## Grade 8

Hold a cube and go close to the mirror. Draw on the lines of the cube on the mirror. What do you see?

How tall a mirror must you buy if you want to be able to see your full vertical image?

Justify your conclusion; try with objects with different distances from mirror; describe ratios in your model

## Grade 12

How tall a mirror must you buy if you want to be able to see your full vertical image? Justify your conclusion; try with objects with different distances from mirror; describe ratios in your model use the cosine rule to derive the height of the actual figure when the height of the mirror image is known

## Grade 13

Draw yourself and a mirror in a three dimensional vector space.

## Lecture or Tutorial task

Given the function $f(x)=x 2-3 x+4$, sketch the function on a pair of axes.
a) Find the equation of a line that crosses this curve where $\mathrm{x}=1$ and $x=2$
b) Find the equation of a line of gradient 3 that crosses the curve twice
c) Find the equation of a line of gradient -3 that does not cross the curve

Use sliders in GeoGebra to determine which of the graphs below could represent the function $y=a x 4+b x 3+c x 2+d x+e$ Here a, b, c, d and e are real numbers, and $a \neq 0$. Explain


## What is the teacher doing ...

... when students work on inquiry-based tasks?

- Circulating and listening
- Asking, and encouraging students to ask questions
- Encouraging dialogue and/or debate
- Fostering reasoning
- Prompting and challenging


## What is the teacher's role?

## Some theoretical ideas

- Conjecturing atmosphere (Mason, Burton \& Stacey, 1982)
- Encouraging students to conjecture and test their conjectures leading to generalisation, abstraction and proof in mathematics
- Teaching triad (Jaworski, 1994)
- Management of learning
- Sensitivity to students
- Mathematical challenge


HARMONY - balance between sensitivity and challenge (Potari \& Jaworski, 2002)

## Inquiry in mathematics

Students engaging in mathematical tasks that are inquiry-based allows

- Multiple directions of inquiry
- Multiple levels of engagement
- Differing degrees of challenge
- Mutual engagement and support
- Harmony in balancing sensitivity and challenge
- Acceptance of and respect for difference


## Turning "I can't" into "I can and I did"

(Open University, 1985)

A teacher set her class the following task:
If a number of circles intersect in a plane, how many regions can be created?

Pupils used hoops on the ground or drew circles on paper or on the whiteboard

1 circle 1 region
2 circles, 3 regions
3 circles, 7 regions
4 circles, 13 regions
$5 \ldots$


## Mary had done something different ...



## What shape is it? (Jaworski, 1988)

- Look at the figure here.
- What is it?
- What shape is it?

- A class of 12 year olds had been asked by their teacher to name the above shape, which he had drawn on the board. Someone said that it was a trapezium. Some students agreed with this, but not all.
- The teacher said, 'If you think it's not a trapezium then what is it?' Michael said, tentatively, 'It's a square ...'
- There were murmurings, giggles, 'a square'?! ... But Michael went on '... sort of flat.


## Contingency

- The teacher looked puzzled, as if re either. He invited Michael to $c$ Michael did.
- He indicated that ya were on your book,


## Responding in the moment to something different (inquiring) --and using it for a good outcome for all

- Then the teacher
- Oooooh yes (!) said the stmoron.... and there were nods around the class. There were e ny


## Inquiry in mathematics teaching

Involves teachers-as-inquirers exploring

- the kinds of tasks that engage students and promote mathematical inquiry
- ways of organising the classroom that enable inquiry activity with all its attributes
- The many issues and tensions that arise related to the classroom, school, parents, educational system, society and politics.
and reflecting on what occurs in the classroom with feedback to future action


## An inquiry cycle

Plan for teaching
Act and observe
Reflect and analyse
Feedback to future planning


Design a particular classroom task

Use the task in the classroom and observe what happens
-- gather data
Reflect on what occurred -- analyse the data

Use what has been learned through observation and analysis to redesign the task
(Jaworski, 2004)

## Inquiry in two layers

# Inquiry in students' mathematical activity in the classroom 

Inquiry in teachers' exploration of classroom approaches

Loughborough University

## Two communities of inquiry

## Inquiry in planning for the classroom

## Inquiry in

 classroom mathematics-= students
(\& teacher)

## Theoretical background: Community of Practice

From Vygotskian theory (e.g., Vygotsky, 1978, Wertsch, 1991)

- Learning as participation in social action mediated by tools and signs

From Wenger’s "Community of Practice" (CoP) (Wenger, 1998)

- Mutual engagement; Joint enterprise; Shared repertoire

Wenger's notion of belonging to a CoP, involves

- Engagement; Imagination; Alignment

BUT Alignment can result in perpetuation of unhelpful practices (e.g., instrumental learning and teaching in mathematics (e.g., Skemp 1985)

## From community of practice ...

Involves

- Engagement
- Imagination
- and
- Alignment

Inquiry

- new kinds of engagement


Critical alignment

- Perpetuation of established practices
(Brown and McIntyre, 1993)

Fibonacci Conference Aptil 2012 -- Inquiry in theory

## Critical Alignment in Practice

We look critically at our practice, while we engage and align with it

- Ask questions about what we are doing and why
- Reveal and question implicit assumptions and expectations
- Try out innovative approaches to explore alternative ways of doing and being to achieve our fundamental goals

Fibonacci Conference Aptil 2012 -- Inquiry in theory

## Creating a Community of Inquiry

## transformattion

## Within a community of practice

- Ask questions and seek answers
- Recognise problems and seek solutions
- Wonder, imagine, invent, explore ...


# Community of inquiry 

- Look critically: ‘critical alignment’ leads to 'metaknowing’
(Jaworski, 2006; 2006; Wells, 1999)


Fibonacci Conference Aptil 2012 -- Inquiry in theory University

- A community of inquiry is central to developmental research in mathematics teaching


## Developmental Research

Research that promotes development as well as charting its progress (Jaworski, 2003)

- Development of teaching (and therefore of learning) in practice. New knowledge in practice
- Development in knowledge at a general level - shareable with others. New knowledge in the wider research community.


## Research is ...

Research is systematic inquiry made public (Stenhouse, 1984)

## A research cycle

Plan/Re-plan

Act \& Observe Reflect \& Analyse
Feedback


Made public Disseminate/Publish

## Developmental Research

Creating and studying development - research that is generative in and transformative of the activity that is researched.

- Engagement in inquiry, within an inquiry community, leads to development of practice (new knowledge in practice) through questioning and critical reflection.
- Engagement in systematic inquiry leads to new knowledge through rigorous analysis of data.
(Jaworski 2006, Goodchild, 2008)
- Critical alignment, looking critically at research practice while engaging in and with it, leads to recognition of issues and ways to address them. (Activity theory analyses)
- Maintaining complexity and fluidity in real settings with all the constraints that institutions and cultures impose.


## What is possible/realistic for teachers?

- Research shows that it is very difficult for teachers to engage alone in action or design research (Eraut 1995; Jaworski, 1994; McIntyre, 1993)
- Teaching is a demanding enough job by itself, without the extra demands of a research role
- Teachers often do not have the specialist knowledge needed to conduct their own research effectively


## Collaborative inquiry

- Two groups of people have important knowledge to bring to teaching development
- Teachers
- Teacher educators/researchers (didacticians)
- Together, these kinds of knowledge can be powerful in the developmental process.


## How is knowledge distributed?

Systemic and cultural settings and boundaries within which learning and teaching are located


Fibonacci Conference April 2012 -- Developmental research

## Co-learning inquiry

In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling.

Both are engaged in action and reflection.
By working together, each might learn something about the world of the other.

Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling.
(Wagner, 1997, p. 16)

## Inquiry in three layers

## Inquiry in mathematics

- Inquiry the classroom
- Inqu Inquiry in mathematics eaching
- Inquiry in addressing a er ons and issues to do with to Inquiry in research
pron
into learning and teaching mathematics


## An example ...

 University
## The 4-year LCM Project in Norway Learning Communities in Mathematics

- Didacticianकी hemethepuniversity + teachers in 8 schools, lower primary to upper secondary
- Minimum of 3 teachers from each school -- School leaders support the project
- 3 years of fieldwork
- Six 3-hour workshops per year at the university, all recorded
- Planning and teaching in school
- Video from classrooms as developmental and research tool.

The design

- Based on inquiry at three levels
- D and T doing mathematics together and discussing didactics and pedagogy (in workshops)
- D designing tasks for workshops
- T modifying tasks for the classroom and/or designing own tasks
- Both D \& T inquiring into own practices insider research
- D conducting outsider research into development in the project as a whole.
(Jaworski, Fuglestad et al, 2007)


Fibonacci Conference April 2012 -- Developmental research University

## Developmental research

- Teachers and didacticians working together sharing knowledge
- Inquiry in 3 layers
- Critical alignment
- Insider and outsider research
- Analysis at two levels
- New knowledge in practice and in the wider research community


## New Knowledge

- Both teachers and didacticians learned in the developmental research process - new knowledge in practice, leading to more knowledgeable ways of working with students and potential for better learning experiences for students.
- Presentations (such as this one) and published papers communicate what has been learned and lead to new knowledge in the wider research community


## What we don't know - yet ...

- We have considerable evidence of the value of these ways of working for development of teaching
- Many questions and issues have been identified and (to some extent) addressed
" One major question is that of "scale" - how can we promote inquiry-based developmental research and practice more widely - a "culture change" is mathematics teaching
(Smith, 2004).


## Thank you for listening

 University
## References

Brown, S., \& Mclntyre, D. (1993). Making sense of teaching. Buckingham, UK: Open University Press.
Eraut, M. (1995). Sch"on shock: A case for reframing reflection-in-action? Teachers andTeaching: Theory and Practice, 1(1), 9-22.
Jaworski, B. (1988). 'Is’ versus 'seeing as'. In D. Pimm (Ed.). Mathematics Teachers and Children. London: Hodder and Stoughton.
Jaworski, B. (1994). Investigating Mathematics Teaching. London: Falmer Press
Jaworski, B. (2003) Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships. Educational Studies in Mathematics 54, 23, 249-282
Jaworski, B. (2004). Insiders and outsiders in mathematics teaching development: the design and study of classroom activity. Research in Mathematics Education, 6, 3-22.
Jaworski B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. Journal of Mathematics Teacher Education, 9(2), 187-211.
Jaworski, B., Fuglestad, A. B., Bjuland, R., Breiteig, T., Goodchild, S., \& Grevholm, B. (Eds.), (2007). Learning communities in mathematics. Bergen, Norway: Caspar.
Jaworski, B., Goodchild, S., Daland, E., \& Eriksen S. (2011). Mediating mathematics teaching development and pu-pils' mathematics learning: the life cycle of a task. In O. Zaslavsky \& P. Sullivan (Eds.), Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning. New York: Springer. Jaworski, B., Fuglestad, A. B., Bjuland, R., Breiteig, T., Goodchild, S., \& Grevholm, B. (Eds.). (2007). Learning communities in mathematics. Bergen, Norway: Caspar.
Jaworski, B. \& Matthews, J. (2011) Developing teaching of mathematics to first year engineering students.

Mason, J., Burton, L. \& Stacey, K. (1982). Thinking Mathematically. London: Addison Wesley
McIntyre, D. (1993). ‘Theory, Theorizing and Reflection in Initial Teacher Education.' In J. Calderhead and P. Gates (Eds.), Conceptualising Reflection in Teacher Development (pp. 39-52). London, Falmer.
Potari, D. \& Jaworski, B. (2002). Tackling complexity in mathematics teaching. Journal of Mathematics Teacher Education, 5, 4, 351-380.

Pring, R (2000). The Philosophy of Educational Research. London: Continuum.
Pring, R. (2004). Philosophy of Education. London: Continuum.
Open University (1985). Working Mathematically with Low Attainers: Centre for Mathematics Education Research Study. Milton Keynes: Open University.
Rowland, T., Huckstep, P. \& Thwaites, A. (2005). Elementary Teachers Mathematics Subject Knowledge. Journal of Mathematics Teacher Education, 8, 3, 255-281

Skemp, R. (1989). Mathematics in the primary school. London: Routledge.
Smith, A. (2004). Making mathematics count. The report of Professor Adrian Smith's inquiry into post-14 mathematics education. London: The Stationery Office.
Stenhouse, L. (1984). Evaluating curriculum evaluation. In C. Adelman (Ed.). The Politics and Ethics of Education. London: Croom Helm.

Vygotsky, L. (1978). Mind in society. Cambridge, MA: Harvard University Press.
Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering research-practitioner cooperation. Educational Researcher, 26(7), 13-22.
Wells, G. (1999). Dialogic inquiry: Toward a sociocultural practice and theory of education. Cambridge, UK: Cambridge University Press.
Wenger, E. (1998). Communities of practice: Learning, meaning and identity. Cambridge, UK: Cambridge University Press.
Wertsch, J. V. (1991). Voices of the mind. Cambridge, MA: Harvard University Press.

