Symmetries and Fibonacci numbers

„Fibonacci phenomena” in theacher training

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Golden section and symmetry in the history of science

Golden section:	From ancient times through the Renaissance to ...
Laws of nature:	17th c.
Symmetry principles: in the form of Variational principles	18th c.
Math. formulation of golden section:	19th c.
Laws and symmetry principles in parallel: (phyisics, crystallography, biology)	19th c.
Decision in favour of Symmetry principles against Laws of nature (physics)	20th c.

„Fibonacci phenomena”

Connection among three things:

- algebraic sequence of numbers
- geometrical interpretation
- figure, denoting
  „the perfect proportion”

The connection among the three phenomena is not unambiguous for the pupils/students

We must explain it.

why symmetry?

Meaning of Symmetry

<Greek>

συν + μετρ(ι)ος

[common measure of things]

↓

συμμετρία

[harmony, proportion]

Allegory of Symmetry

D. Calvaert (1540–1619), Bologna

(Museum of Fine Arts, Budapest graphics collection,, K.66.25.)

Symmetry phenomena

Reflection

Rotation

Translation

Glide reflection

Similitude

Affine projection

Topological symmetries

Permutational symmetries

...

+

+

Why do we call all of them symmetries?

What are the common in the above listed phenomena?

They are so much different phenomena (and we have not listed all forms of symmetry).

Why do we call all of them symmetries?

Geometrical concept of symmetry

– In each instance we performed some kind of (geometrical) *operation* (transformation).

– In this process, *one or more* (geometrical) *characteristics* (properties) *of*

– *the* (geometrical) *figure* (object) remained unchanged.

– This characteristics proved to be *invariant under the given transformation* (did not change as a result of the operation performed).

Generalization of the concept of symmetry (1)

From
harmony
to

perfection,
beauty,
truth,
mean

Platonic –
Aristotelian
generalisation:

Generalization of the concept of symmetry (2)

- not only for *geometrical* operations
  and
- not only for *geometrical* objects,
  and
- not just for *geometrical* characteristics

We generalise the geometrical meaning of symmetry in such a way that the interpretation be valid

Generalization of the concept of symmetry (3)

In a generalised sense, we can speak of symmetry if

– *in the course of any kind of* (not necessarily geometrical) *transformation* (operation)

– *at least one* (not necessarily geometrical) *characteristics of*

– *the affected* (arbitrary and not necessarily geometrical) *object*

***remains invariant* (unchanged).**

Generalization of the concept of symmetry (4)

The generalization
took place
with reference
to three things:

- to any transformation,
- to any object,
- to any characteristics.

Why are the Fibonacci phenomena symmetries?

(1) Greek interpretation:

harmony,

the perfect proportion

Why are the Fibonacci phenomena symmetries?

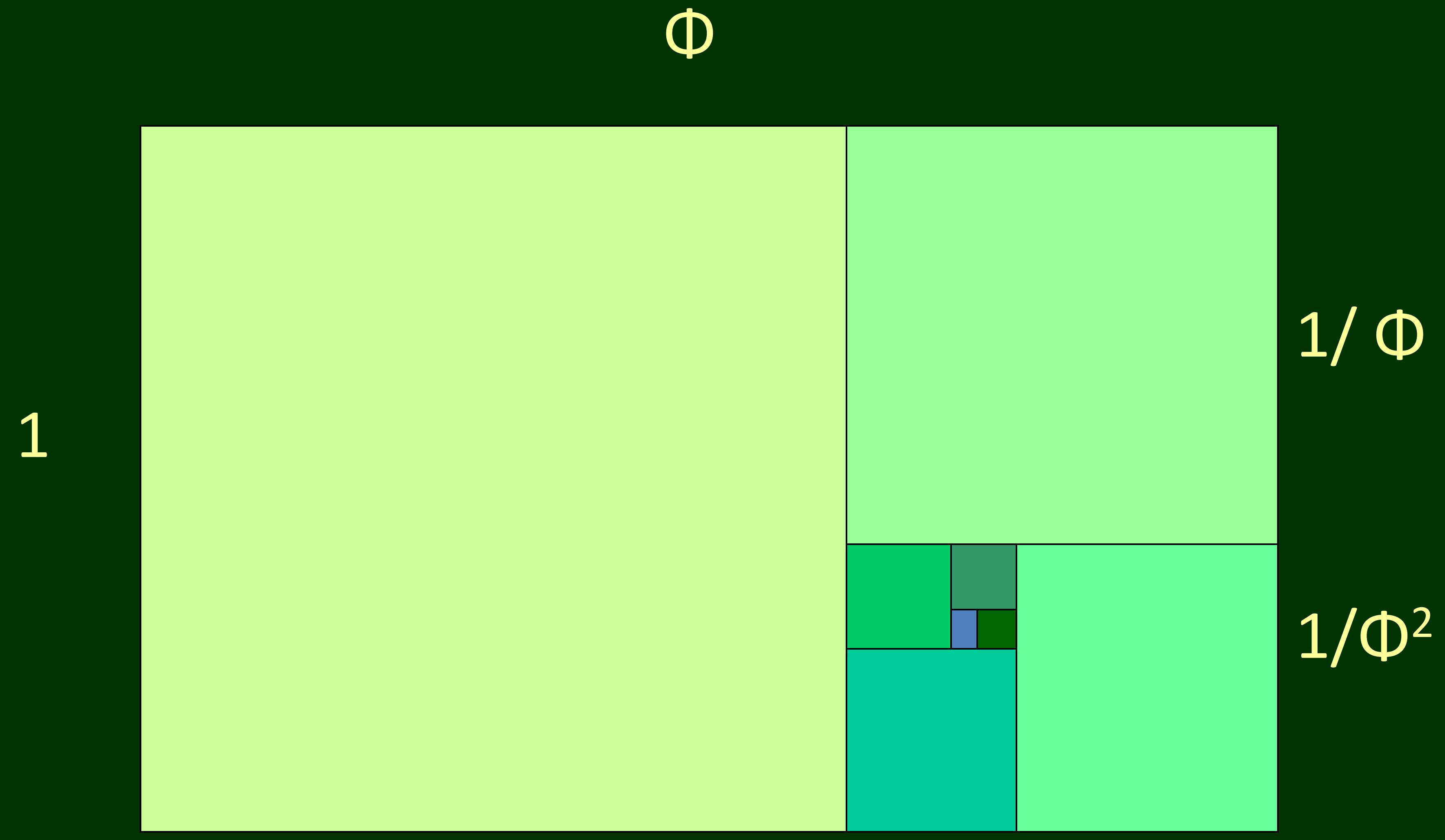
(2a) algebra: $a_n = a_{n-1} + a_{n-2}$

- repetitive rule of construction (algorhytm)

(2b) Geometrical interpretation:

- invariance under similitude transformation

Golden rectangle



Why are the Fibonacci phenomena symmetries?

(3) Generalised interpretation

E.g., quasicrystals,
proportion of Penrose tiles,
genetic matrices,
fractal structure of ancient Mesoamerican art , ...

The place of „Fibonacci phenomena” in shaping *interdiscplinary approach* and *holistic world view* of students in teacher training

Course annotation:

http://hps.elte.hu/oktaeder/atmeneti/darvas.htm#English

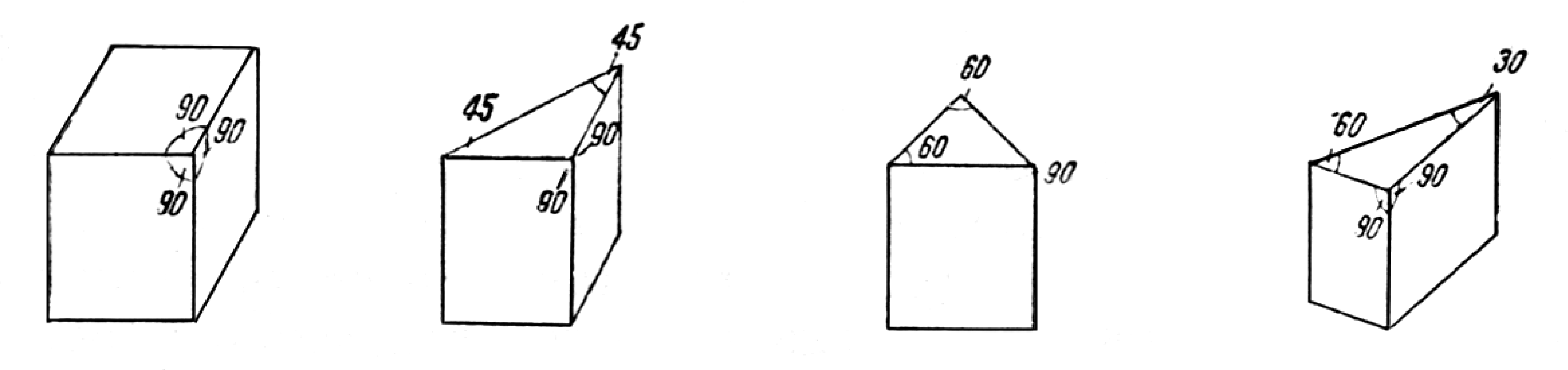
Textbook/Monograph on *Symmetry* (Birkhäuser, 2007)

Why symmetry- and Fibonacci phenomena in teacher training?

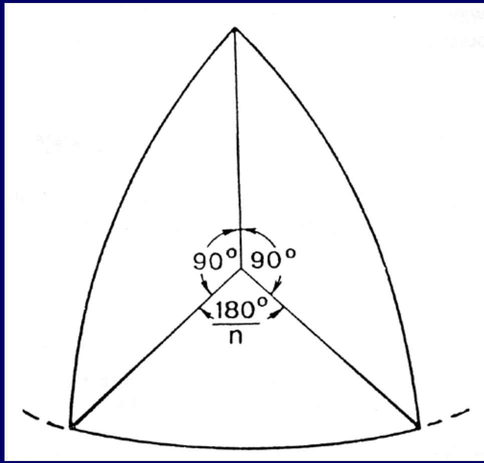
They are exceptionally suitable for illustration:

- to include geometrical practices
  (e.g., golden section kaleidoscope,
  proportions in the perfect solids, ...)

Prismatic kaleidoscopes

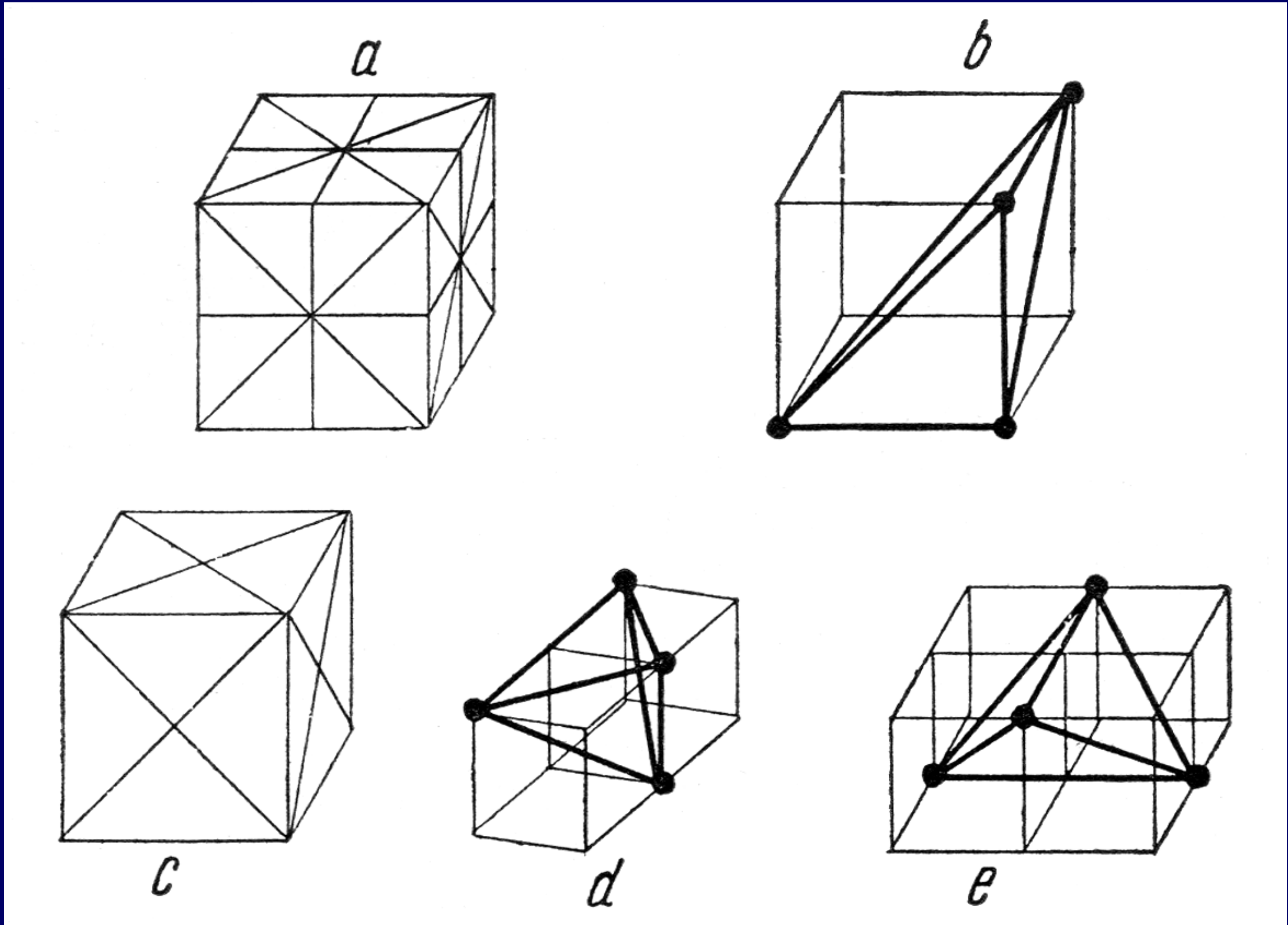


Angles of Fedorov-type kaleidoscopes:

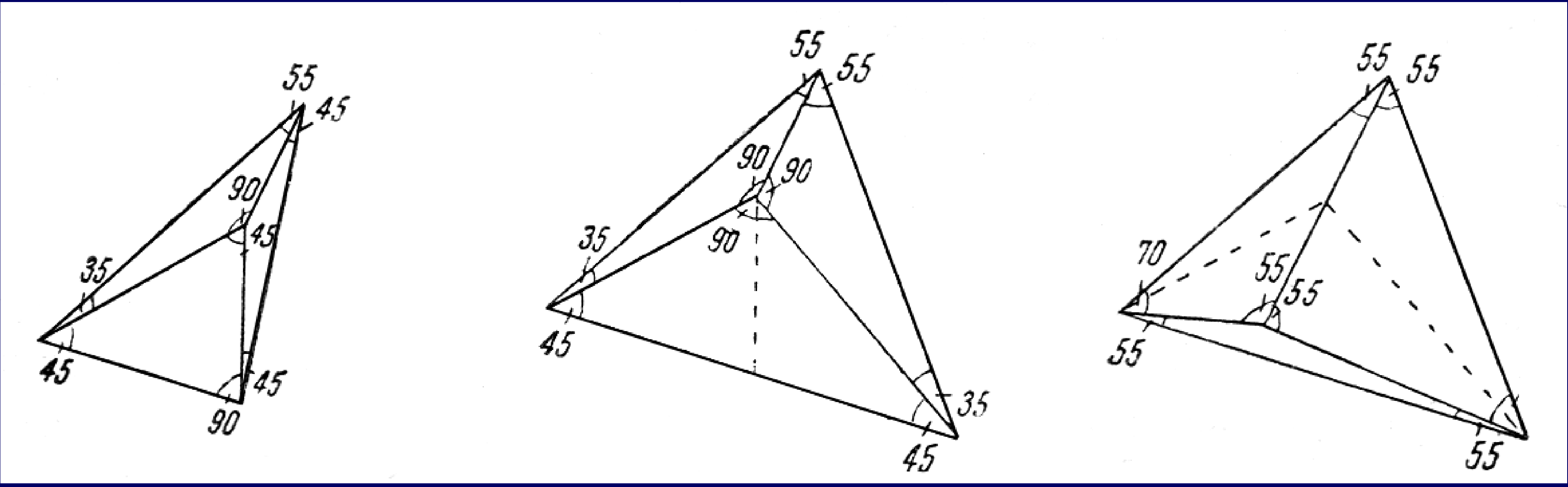


Tetrahedral kaleidoscopes

Producing kaleidoscopes by breaking up a cube:

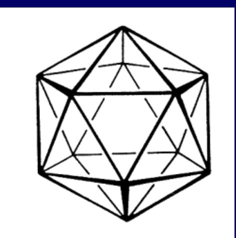


Tetrahedral kaleidoscopes

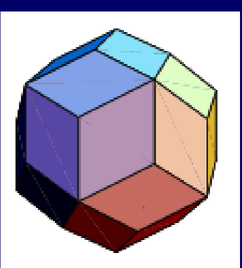


Golden section kaleidoscope

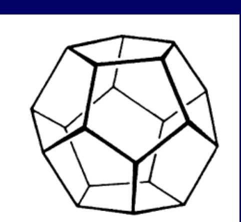
(Angles: 32°, 37°, 21°)



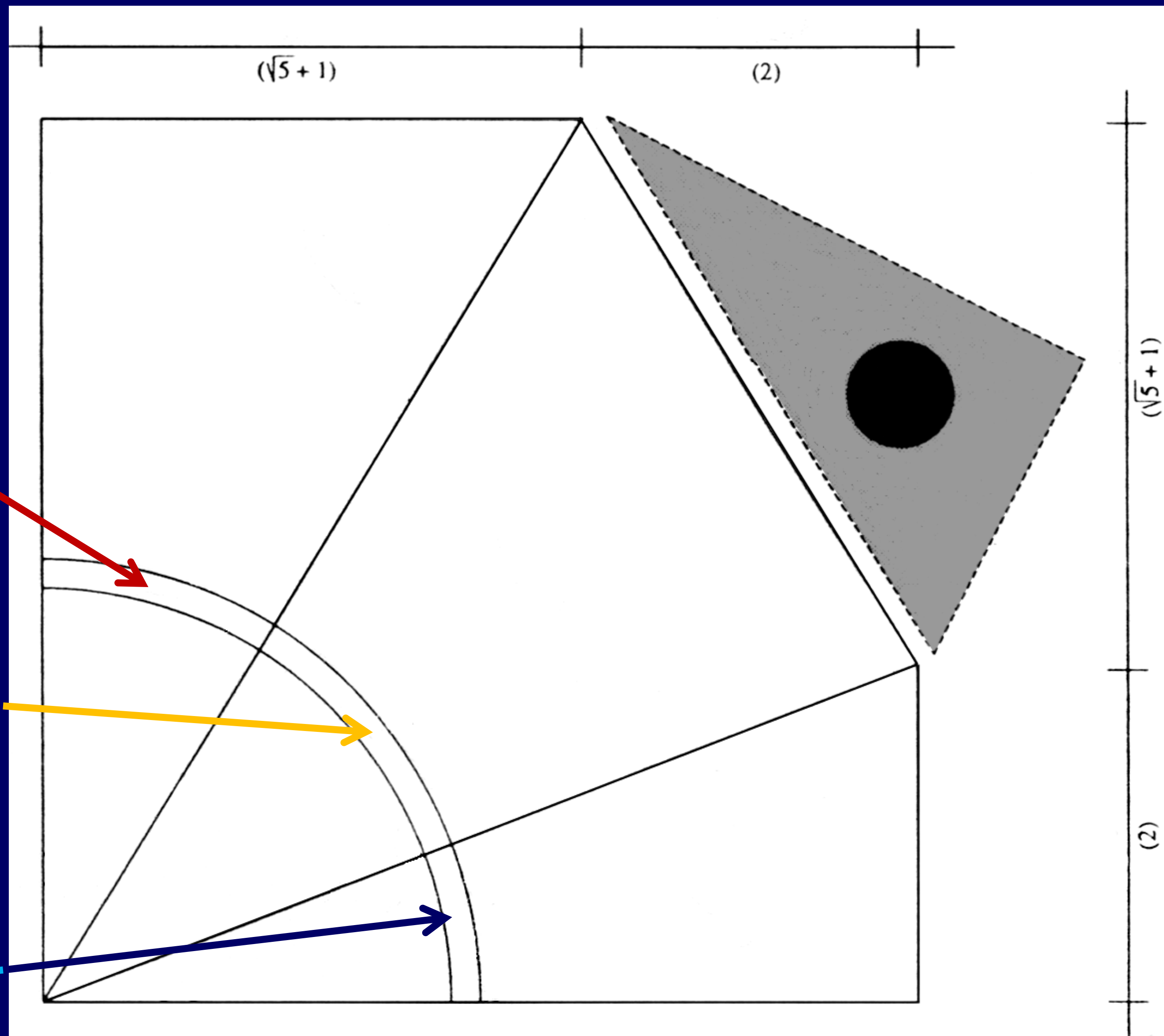
Icosahedron



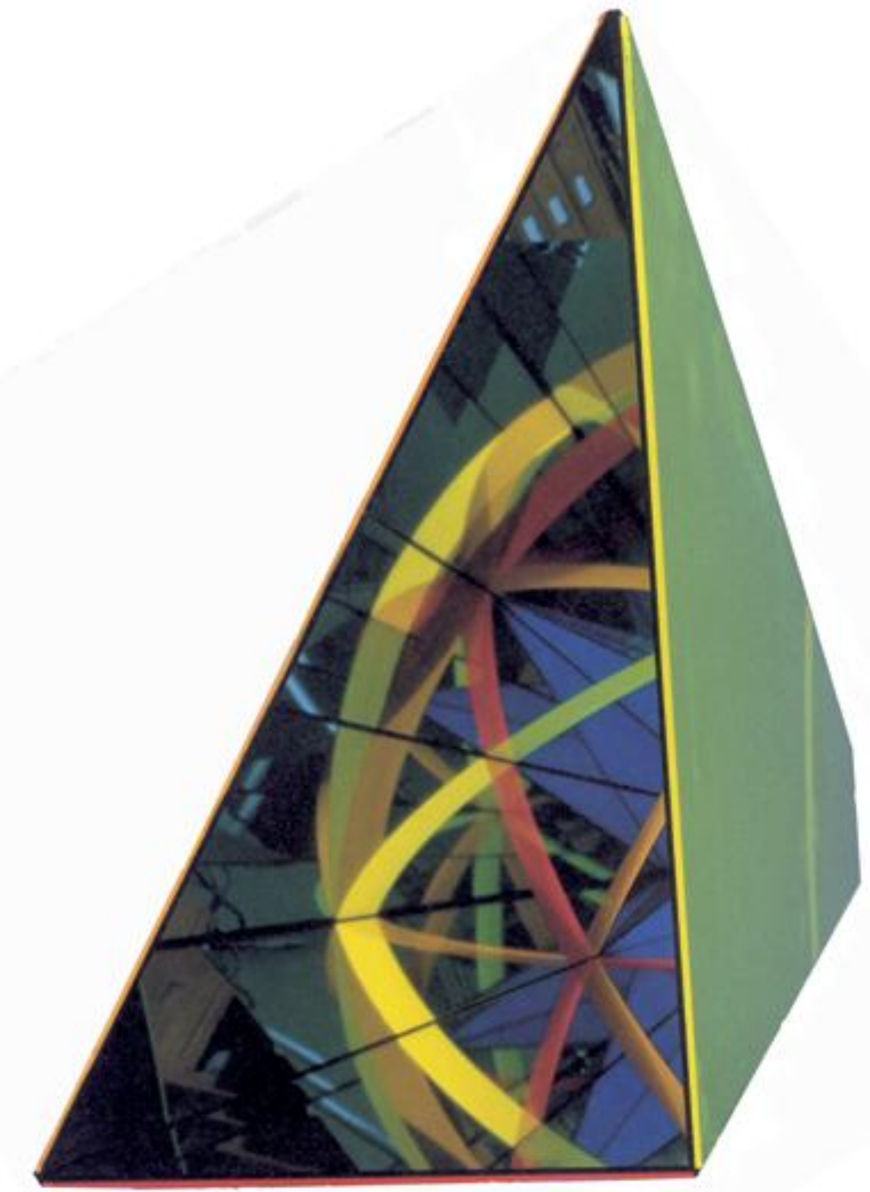
Rhombic Triacontahedron



Dodecahedron



Golden section kaleidoscope



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  proportions in the perfect solids, ...) ;
- to bring artistic examples in the math classroom

Why symmetry- and Fibonacci phenomena in teacher training?

They are exceptionally suitable for illustration:

- to include geometrical practices (e.g., golden section kaleidoscope, proportions in the perfect solids, ...) ;
- to bring artistic examples in the math classroom;
- to show their many interesting algebraic properties, what
  - - make the students like to learn mathematics (incl. algebra, geometry, trigonometry, analysis, numeric theory)
  - - to seek for further symmetries (in sciences, in arts)

Course thematics

INTRODUCTORY LECTURES

INTERDISCIPLINARY EXAMPLES

APPLICATIONS IN THE PHYSICAL NATURE

BRIDGES TO THE MAN

BRIDGES TO THE HUMANITIES

(1) INTRODUCTORY LECTURES

1. The concept of symmetry, invariance, harmony.

2. Historic background.

3. Frieze patterns (groups), wallpaper patterns (groups), crystallographic groups. Symmetry in decorative art, space groups, crystal structures.

4. Golden section. Fibonacci sequences.

(2) INTERDISCIPLINARY EXAMPLES

5. The harmony of the built environment. Phillotaxis in the organic world.

6. The perfect solids: from Plato to the crystals.

7. That mysterious fivefold symmetry: from Dürer to the quasicrystals.

8. From the structure of viruses, through stability of built structures, to the Fullerene molecules.

(3) APPLICATIONS IN THE PHYSICAL NATURE

9. Cosmological symmetries.

10. Seeing and hearing: the harmony and physics of the world of colours and tones.

11. Generalisation of the concept of symmetry in physics. Symmetry breaking in the inanimate nature.

(4) BRIDGES TO THE MAN

12. Chirality. Morphological and functional symmetry breaking along the evolution of the organic matter.

13. Asymmetries of the human brain and its consequences. Symmetry in mathematical and logical thinking.

(5) BRIDGES TO THE HUMANITIES

14. The beauty and the truth. The emotional and rational functions of the human brain: arts, techné, science.

15. Rationality and impression: function and art in the works of art and technology in the 20th century.

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Textbook/Monograph on *Symmetry* (Birkhäuser, 2007)

Symmetry Festival 2006: *Symmetry in Education*

Danke schön

für Ihre

Aufmerksamkeit