Mathematical Education meets Reality and Future

Prof. Dr. Ulrich Trottenberg, Director
Fraunhofer Institute for Algorithms and Scientific Computing SCAI
and Mathematical Institute, University of Cologne

Conference for “Raising Awareness about Inquiry Based Science and Mathematics Education (IBSME) in Europe”

Plenary Session “Relevance of mathematics and science education for society”

Bayreuth, September 21st, 2010

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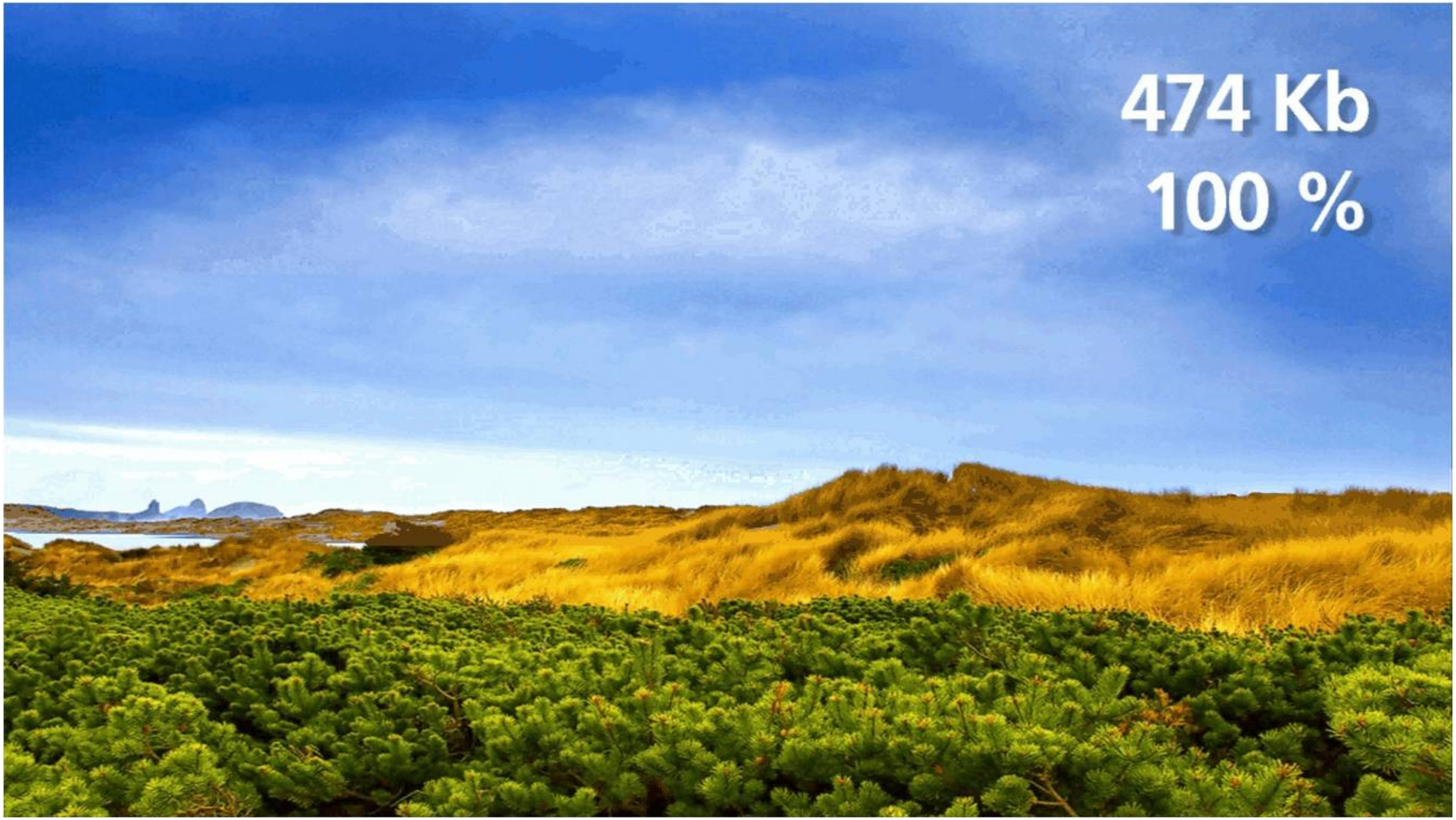
Data Compression: JPEG and MP3



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The Image of mathematics (in Germany)

- difficult
- boring
- detached from reality

somewhat different in other countries

- France, Italy, … theory oriented
- Scandinavia application oriented

- Asia seen as an intellectual playing field
- Anglo-America
- Latin America
-

The “MINT” problem

More than 100,000

- mathematicians
- computer scientists
- physicists, chemists, …
- engineers

missing in Germany

One reason: reservation against mathematics

Consequences, remedies

Change the

- image of
- perception of
- interest in

mathematics (in Germany)

Change

mathematical education

Mathematics is both: abstraction and application

Breakthroughs in Pure Mathematics

- Four-Colour problem
- Mordell‘s conjecture
- Fermat‘s last theorem
- Poincare‘s conjecture

Impact on mathematical education?

Mathematics is both: abstraction and application

- Applied Mathematics has drastically changed over the last 50 years:
  - Penetration of science, technology and economy
  - Basis for all technical developments
  - Modeling, simulation, optimization, …
- Applied Mathematics in society and everyday life:
  MP3, mobile phone, GPS navigation, online banking, search engines, …
- Revolution through computer development:
  algorithms replace formulae

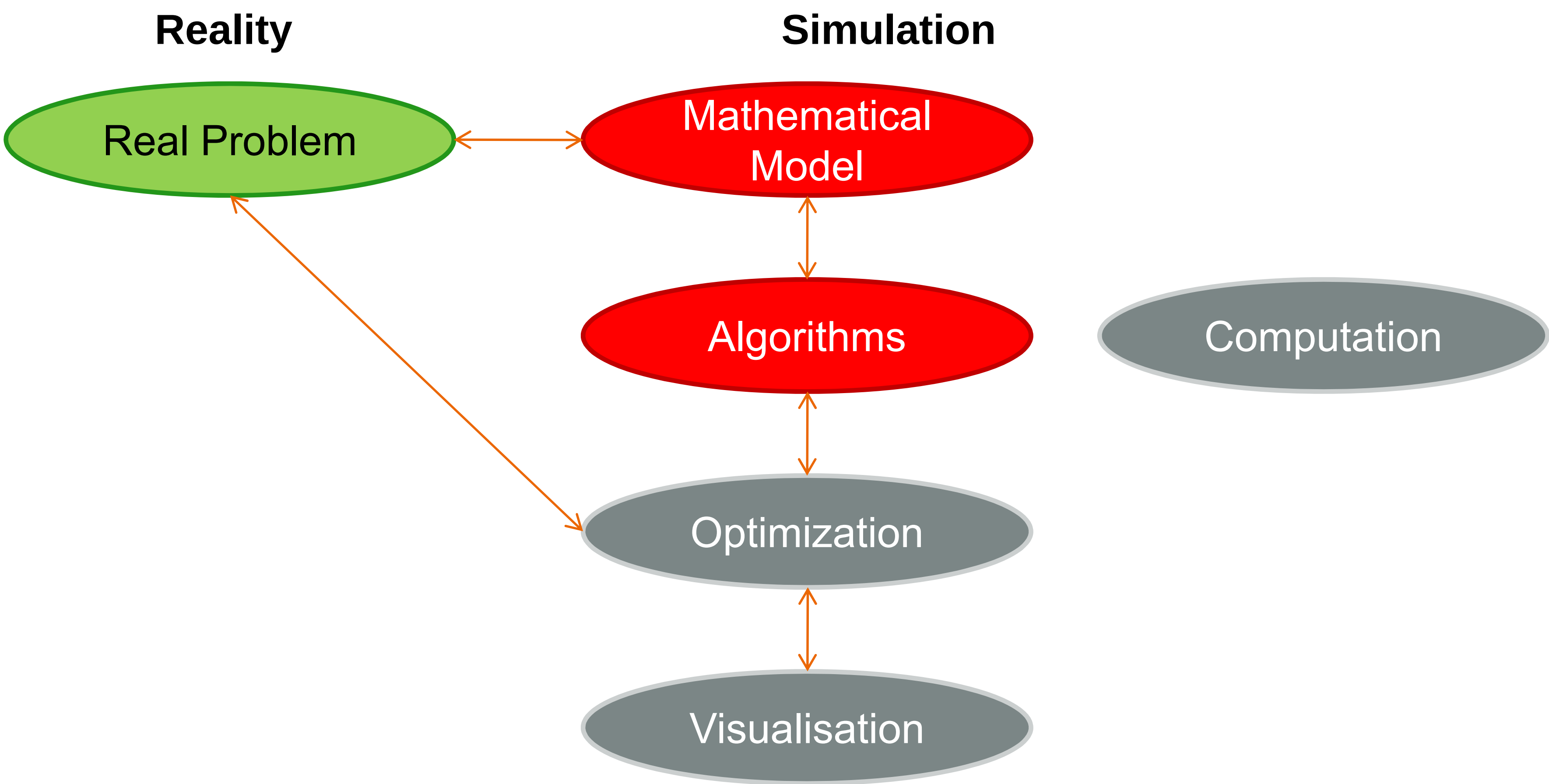
Applied Mathematics in mathematical education

Applied Mathematics revolution

has little (no) impact on mathematical education, so far

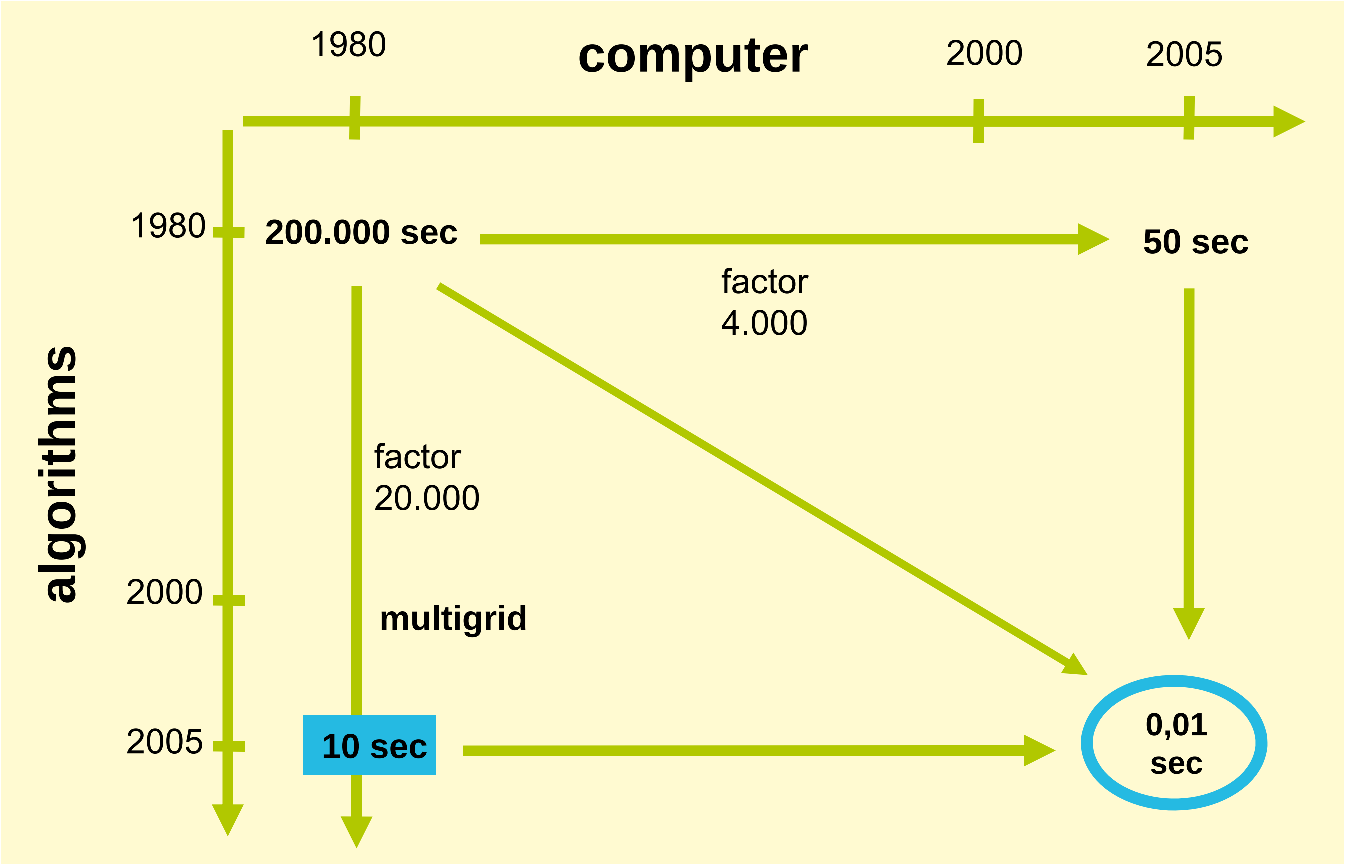
Many teachers are not aware of this revolution

Applied Mathematics today



Breakthroughs in Applied Mathematics: algorithms

Algorithms versus hardware



Example:
convection/diffusion
equation

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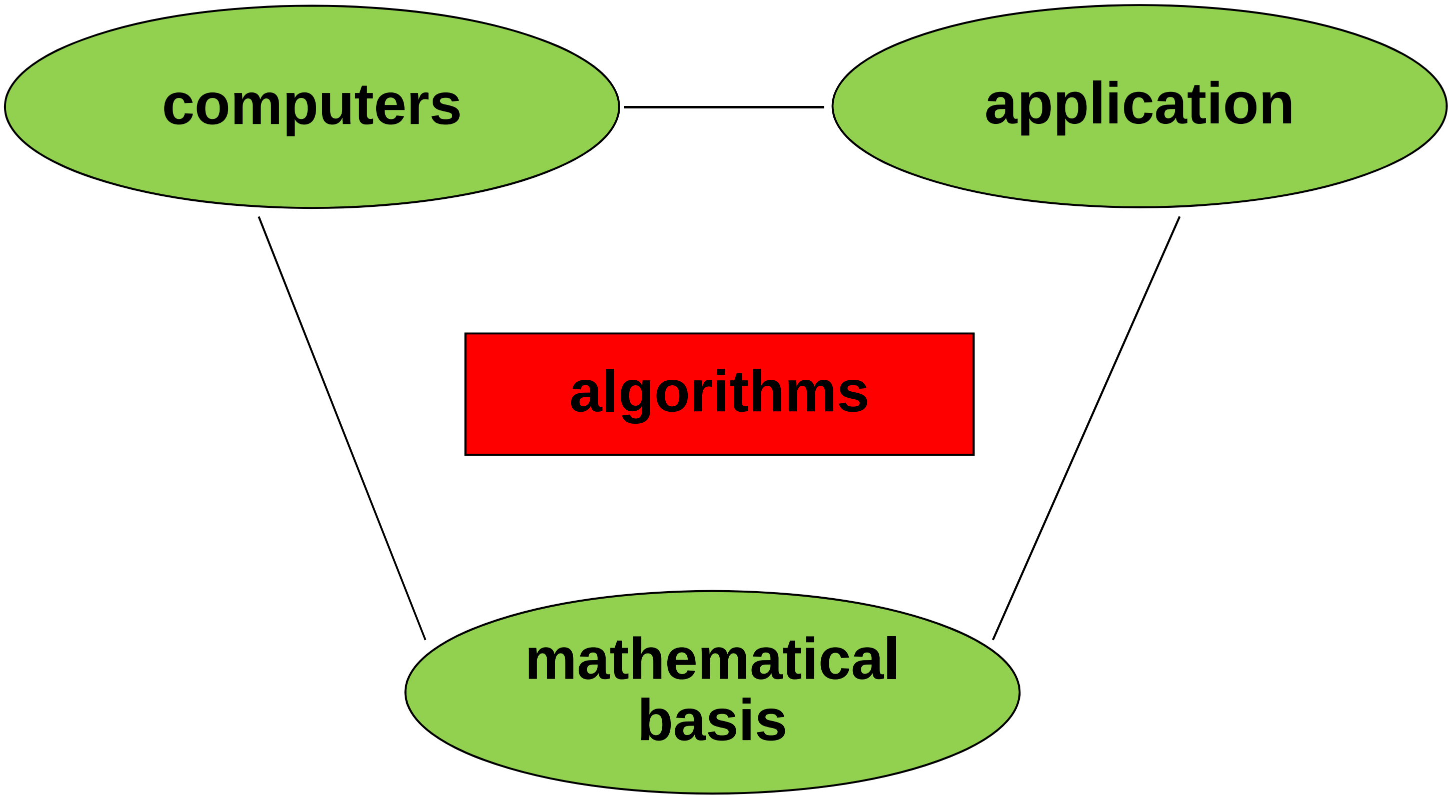
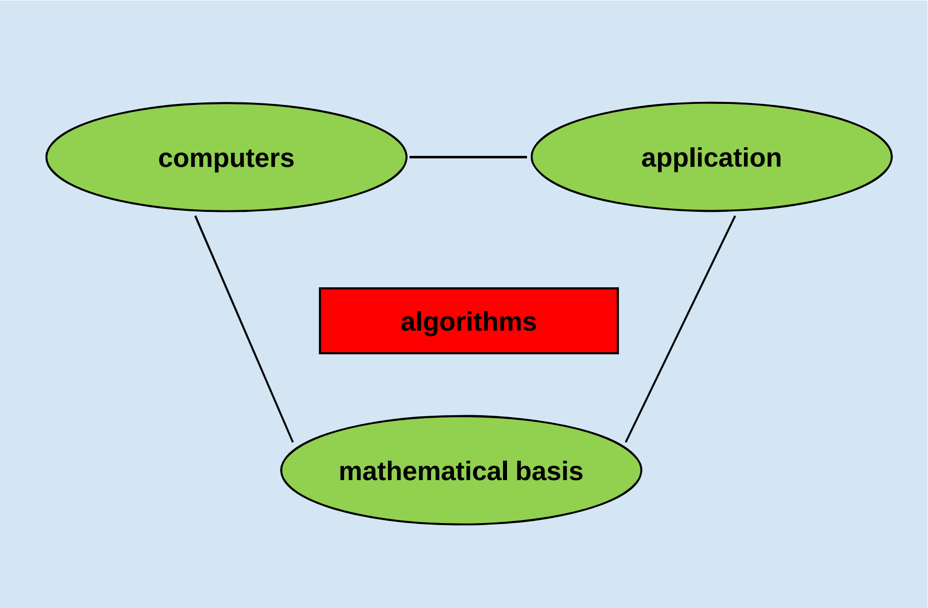
Idea: Bring algorithms to school



Algorithms in mathematical education

Algorithms are everywhere today:

- MP3
- mobile phone
- GPS navigation
- online banking
- search engines
-



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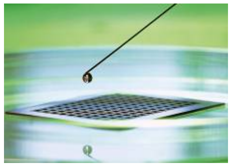
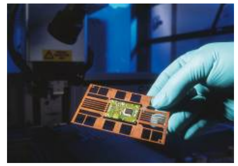
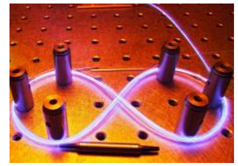
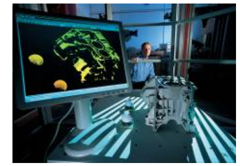
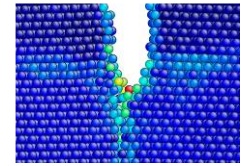
The Fraunhofer* - Gesellschaft

- applied and industrially oriented research, of wide benefit to society
- founded in 1949
- annual budget 1.6 billion Euro

  approx. 40% industry funding
  40% project funding
  20% institutional funding
- international research centers
- 17,000 employees
- 59 R&D institutes

*** Joseph von Fraunhofer (1787-1826)**
Researcher, Inventor, Entrepreneur

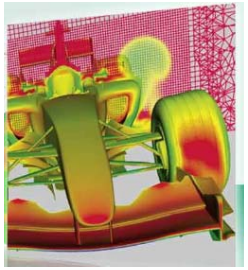
The Profile of the Fraunhofer-Gesellschaft


- Mathematics and Information Technology
- Microelectronics
- Production
- Materials and Components
- Light & Surfaces
- Life Sciences
- Defense and Security

Applied Mathematics in Fraunhofer
Mission: Innovation in technology, economy, life science, society

- Fraunhofer SCAI
  Algorithms and Scientific Computing
  Sankt Augustin, Cologne, Bonn
  Algorithms

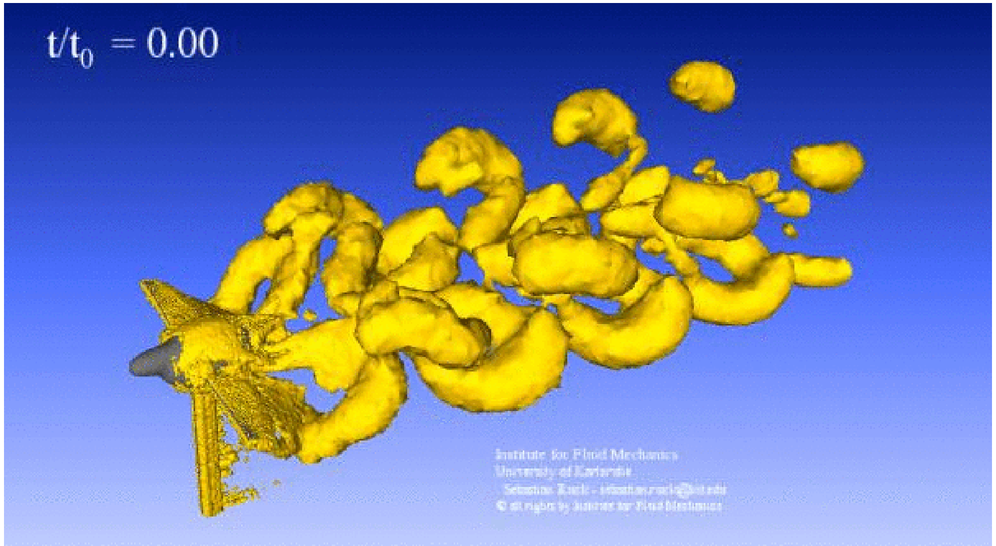


- Fraunhofer ITWM
  Industrial Mathematics
  Kaiserslautern
  Modeling

- Fraunhofer MEVIS
  Medical Image Computing
  Bremen
  Visualization

Applied Mathematics today

„Multiphysics“, e.g. fluid-structure-interaction:
MpCCI (mathematical software product)

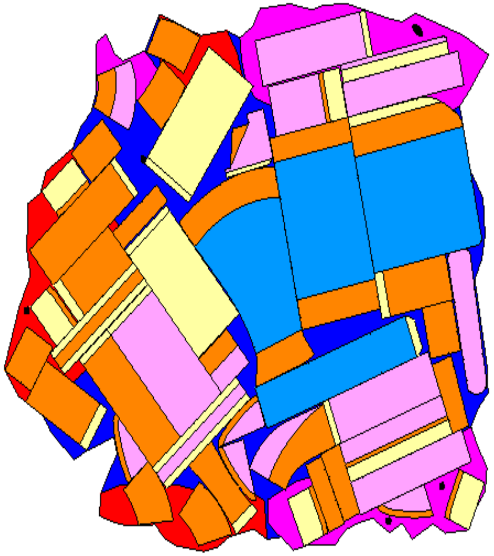


2010

Tacoma Narrows-Bridge
1940

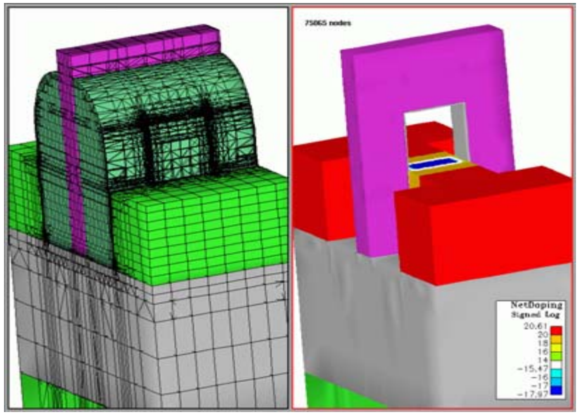
Applied Mathematics today

Optimal cutting:
AutoNester (mathematical software product)



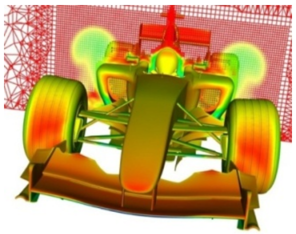
Applied Mathematics today

Simulation => large linear systems

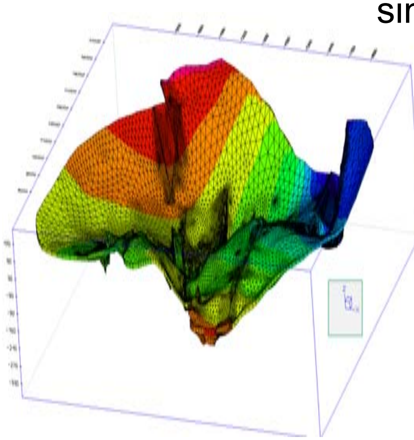


semiconductor device simulation

computational fluid dynamics

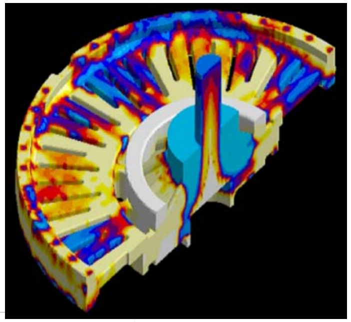


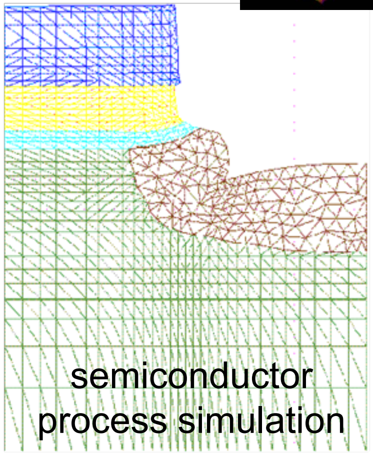
oil reservoir simulation



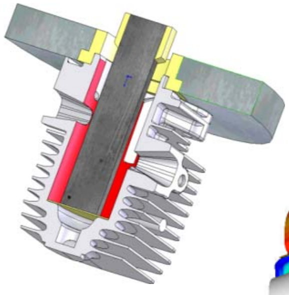
ground water simulation, geo- physics

structure mechanics

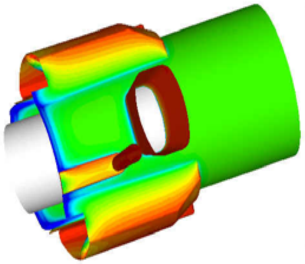




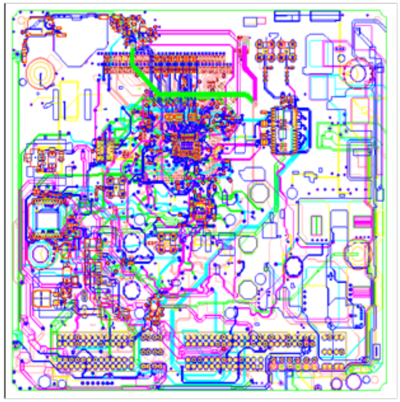
semiconductor process simulation



electrochemistry



circuit simulation, electromagnetic compatibility



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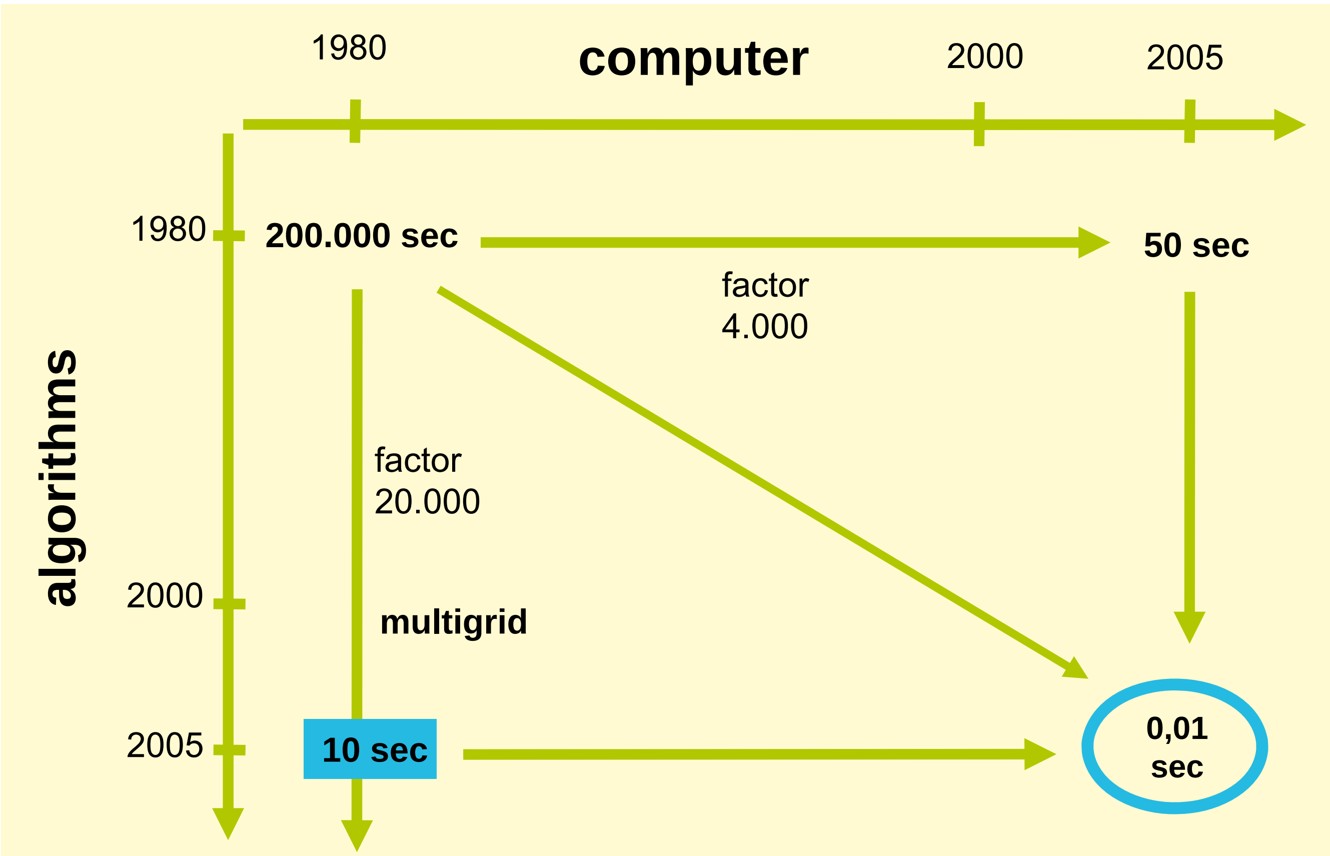
3. Why algorithms – examples
4. Consequences, recommendations

Algorithms versus hardware



Example:
convection/diffusion
equation

Fast linear solvers (for large linear systems)

$$\boxed{\sum_{j=1}^{N} a_{ij} x_j = b_i \quad (i = 1, \ldots, N)}$$

N=3

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned} \qquad \begin{aligned} 8x_1 - 3x_2 + 3x_3 &= 1 \\ 7x_1 + 9x_2 + 2x_3 &= 1 \\ 2x_1 + 3x_2 - 5x_3 &= 1 \end{aligned}$$

N in real applications: ≈ 100 millions

Fast linear solvers

Example: weather forecast

Euler equations

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

$$\frac{\partial}{\partial t}(\rho u)+\rho u^2+\frac{\partial}{\partial x}p+\frac{\partial}{\partial y}(\rho uv)=0$$

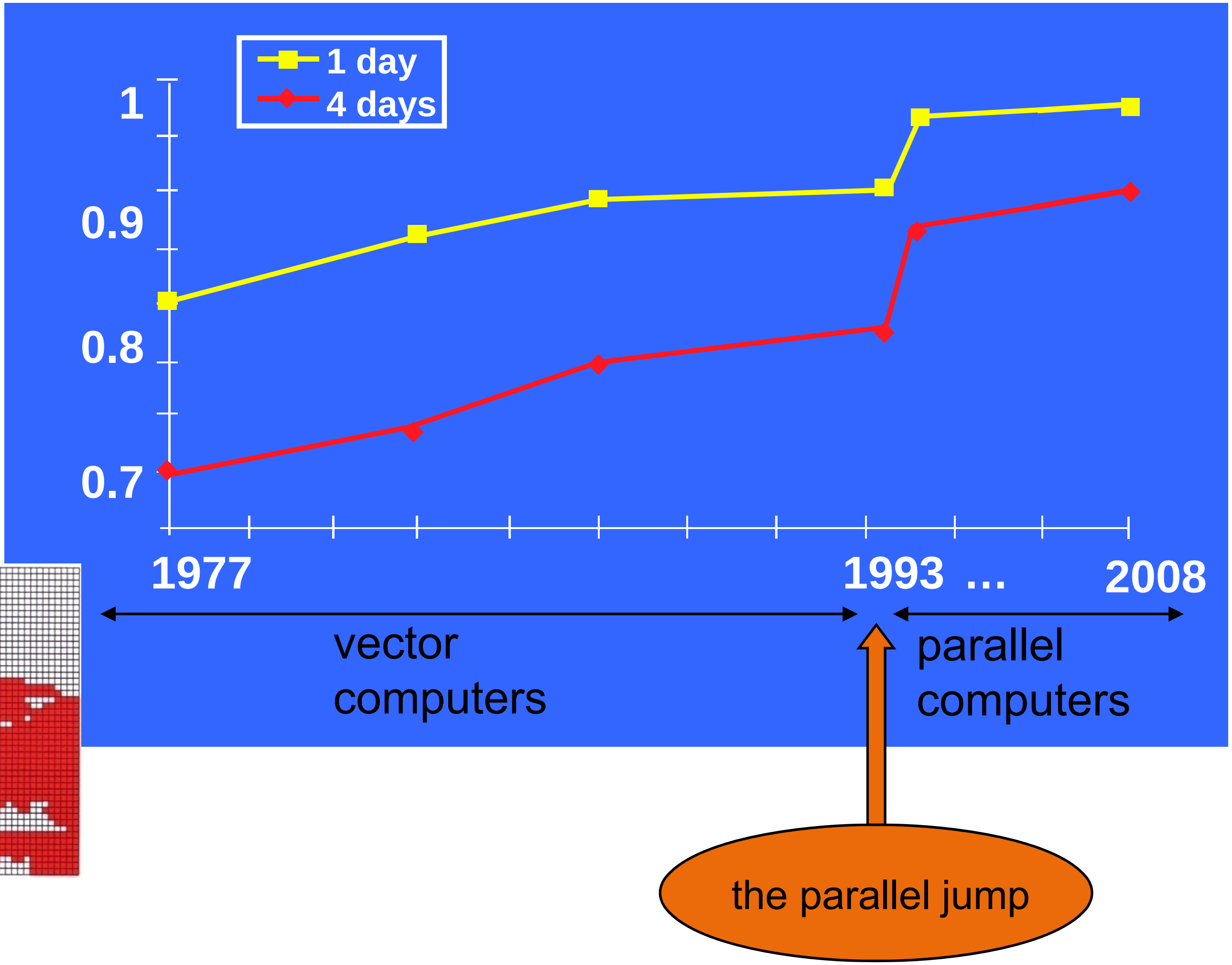
$$\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}(\rho v^2+p)=0$$

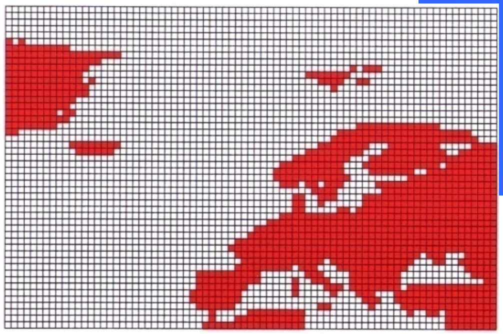
$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial x}((E+p)u)+\frac{\partial}{\partial y}((E+p)v)=0$$

$$p-(\gamma-1)\left(E-\frac{1}{2}\rho\left(u^2+v^2\right)\right)=0$$

Fast linear solvers

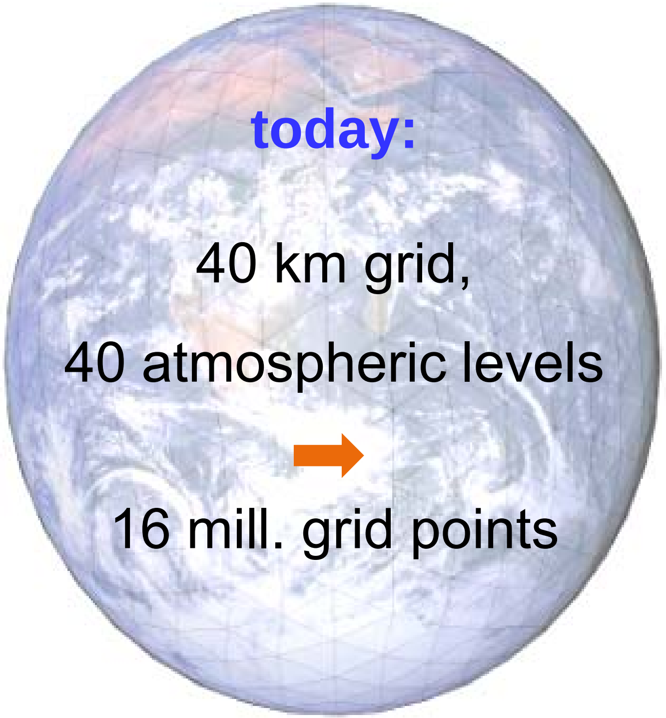
Quality of weather forecast





120 km

Fast linear solvers



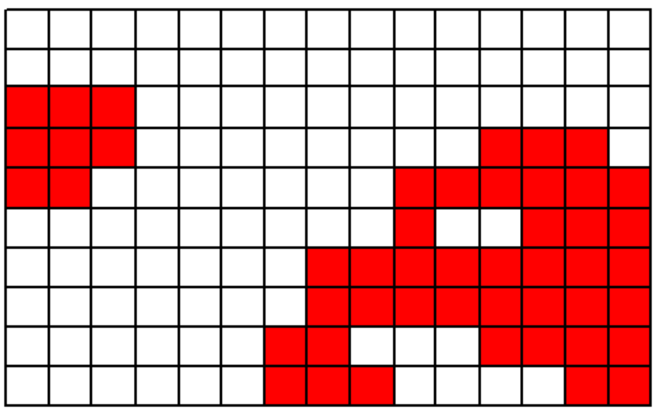
to solve

linear systems with approximately 16 mill. unknowns (in each time step)

approximately 6.500 time steps for 10 days forecast

Fast linear solvers

Climate prediction



EUROPE in a 600 km grid

time scale:	100 years (instead of 10 days)
resolution:	???
computing time:	months
(in)accuracy:	2° - 5° C uncertainty (for global warming)

Efficiency of some linear solvers

Example: 2D-Poisson-equation, discretized on 1024 x 1024 grid
~> linear system with about 1 million unknowns

Solver	Number of operations	Computing time on standard PC	Factor to MG
Gaussian elimination for band matrices	$\sim N^2$	14 h	50000
SOR	$\sim N^{1.5}$	5 min	300
Multigrid	$\sim N$	1 sec	1

Data compression: MP3

Data compression: MP3

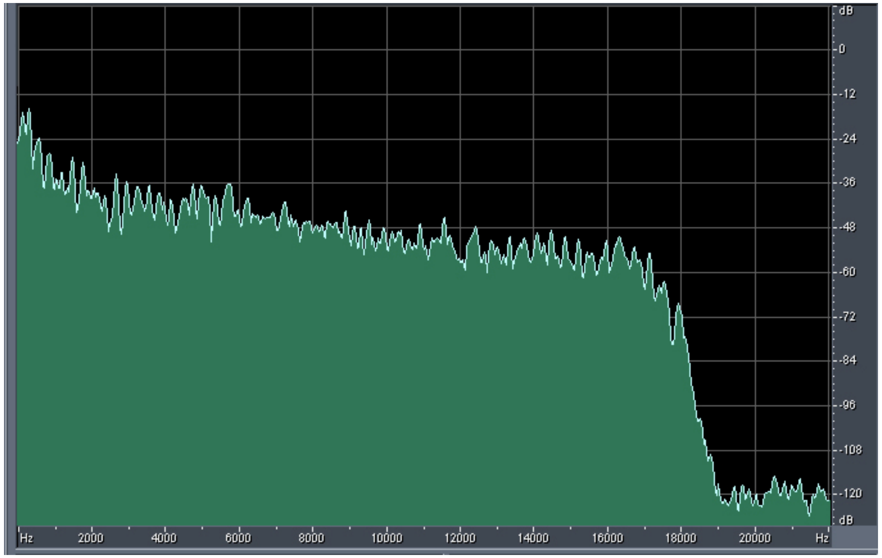
mathematics … music

Fourier-transformation:

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i2\pi ft} dt$$

Discrete Fourier-transformation:

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-i2\pi kn/N}, k = 0,\ldots, N-1$$



(Source: Fraunhofer IIS and ITWM)

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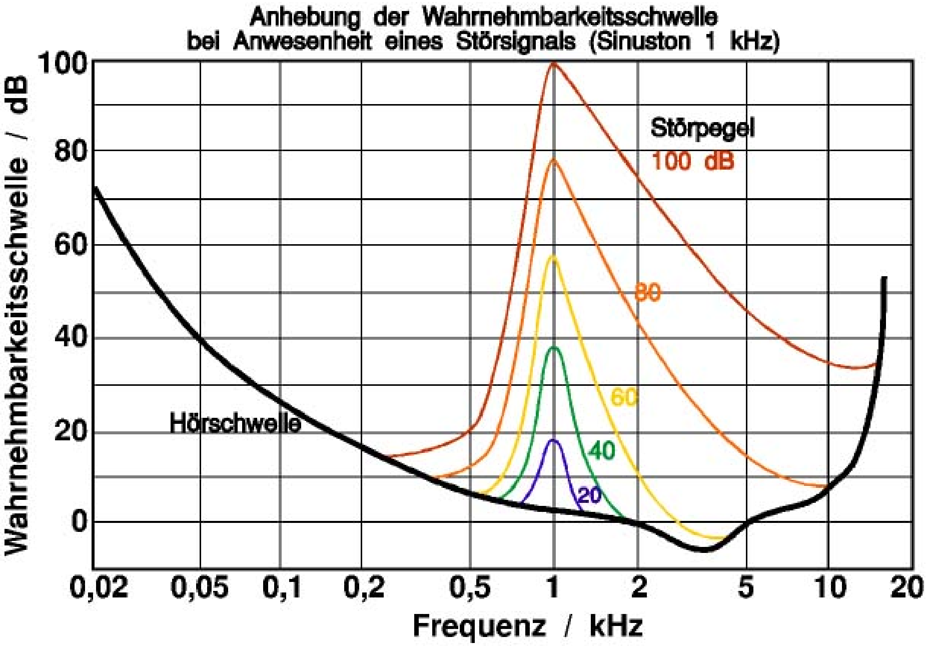
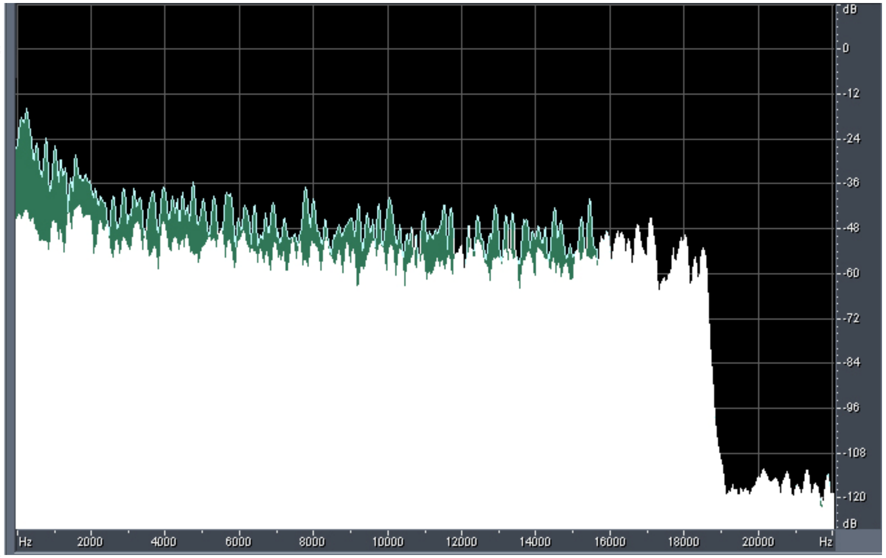
Data compression: MP3 - „Quantification“

mathematics … music





(Source: Fraunhofer IIS and ITWM)

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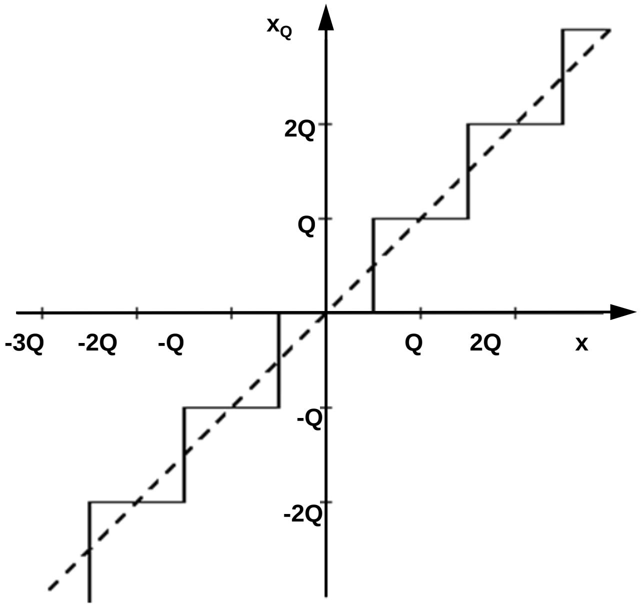
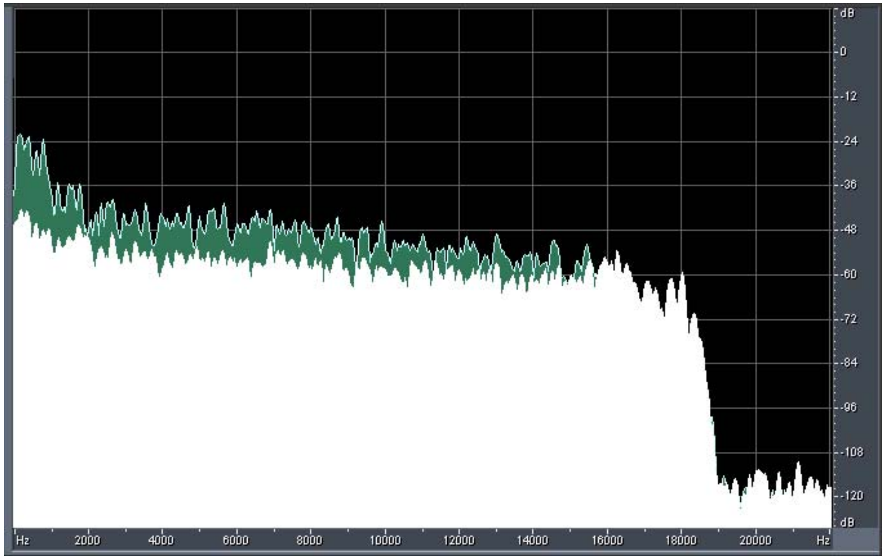
Data compression: MP3 - What can we hear?

mathematics … music





(Source: Fraunhofer IIS and ITWM)

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Data compression - store differences

How to store data, like

993, 1003, 1008, 997, 1004, 995, 1003, 997, 1001, 991, 997

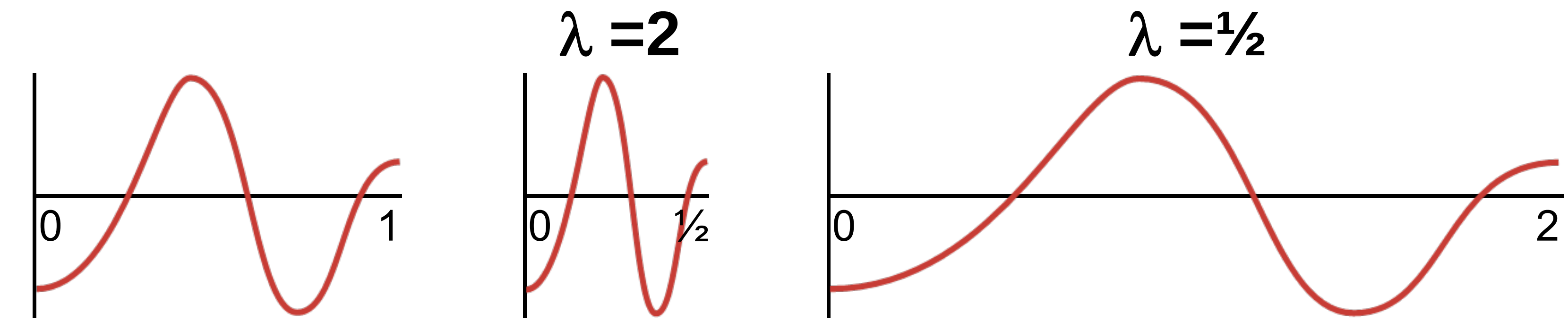
efficiently?

Idea: store **differences** from 1000 (smaller numbers!)

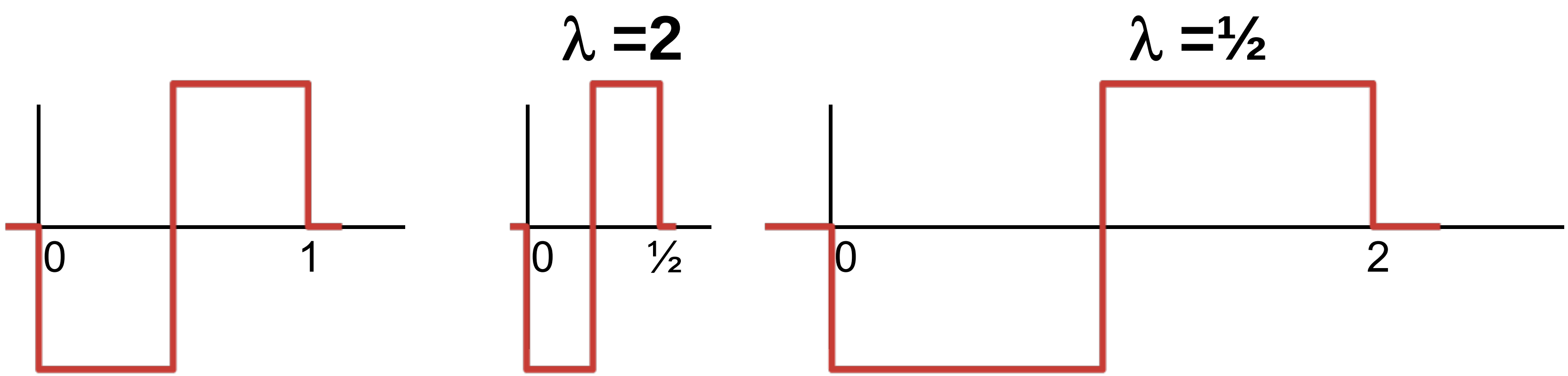
-7, +3, +8, -3, +4, -5, +3, -3, +1, -9, -3, …

(Simplified) MP3 principle: **differences** from mean values (recursively)

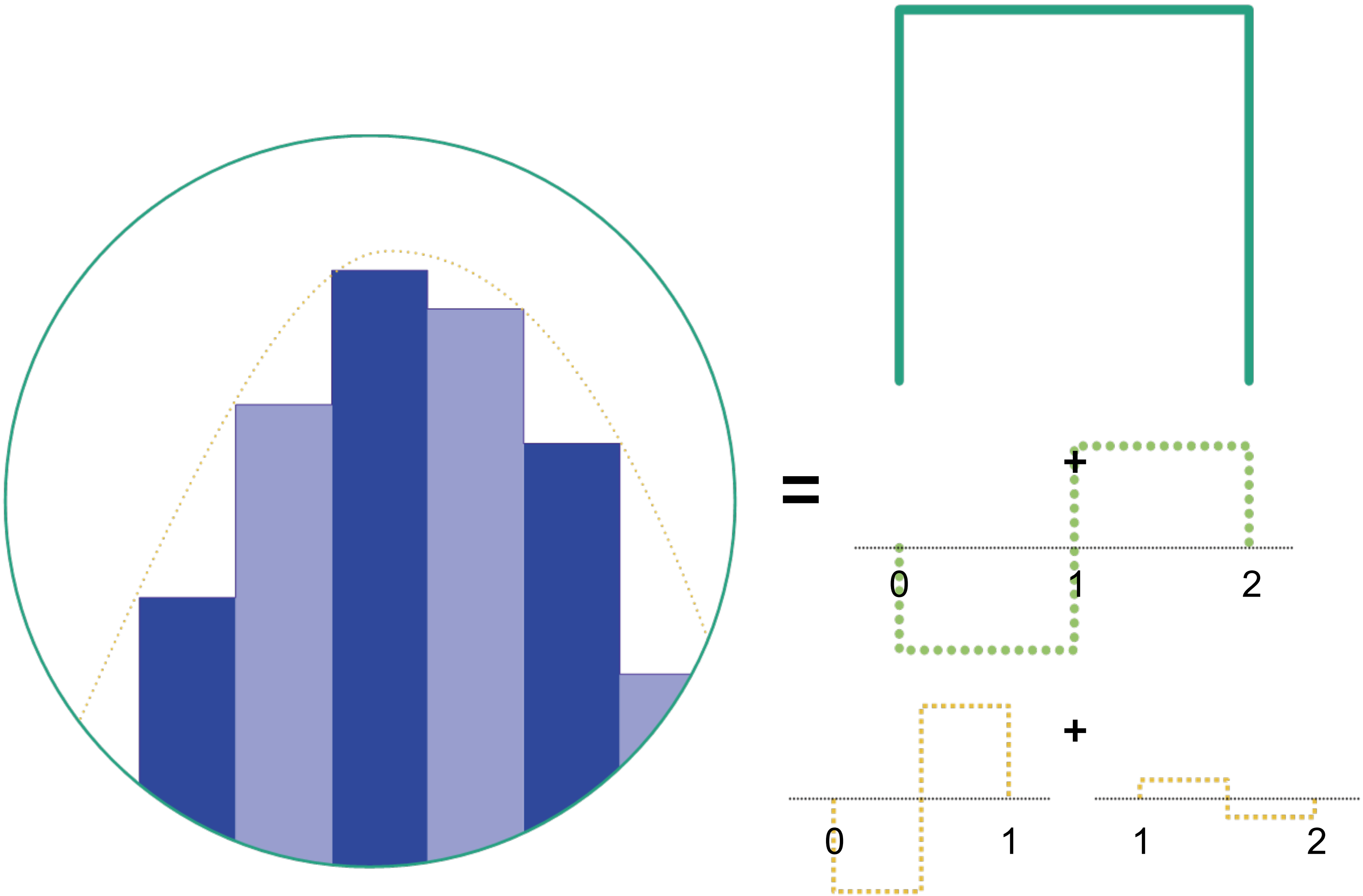
Recursive multi-scaling: f(x) => f(λ x)



Fourier



Wavelet



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Algorithms in mathematical education: modules

- MP3
- Fast linear solvers
- Tour planning
- Traffic simulation
- Cryptography (RSA-algorithm, prime numbers)

Development funded by WestLB Stiftung Zukunft NRW

Modules

- MP3
- Fast linear solvers
- Tour planning
- Traffic simulation
- Cryptography (RSA-algorithm, prime numbers)
- GPS navigation
- Newton‘s method and bisection
- Sorting and searching
- Linear programming
- Chaos and fractals
- Exponential and logistic growth

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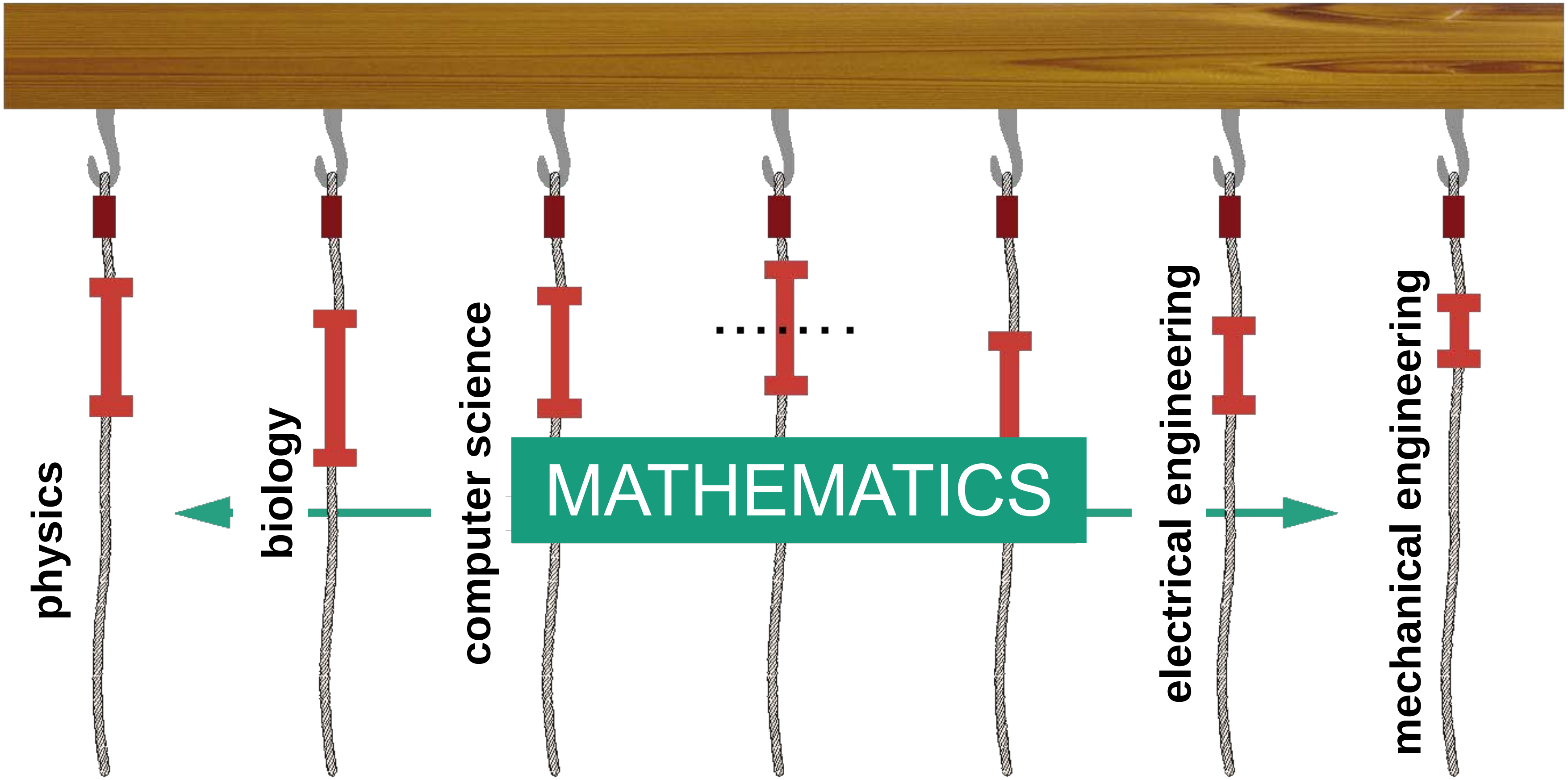
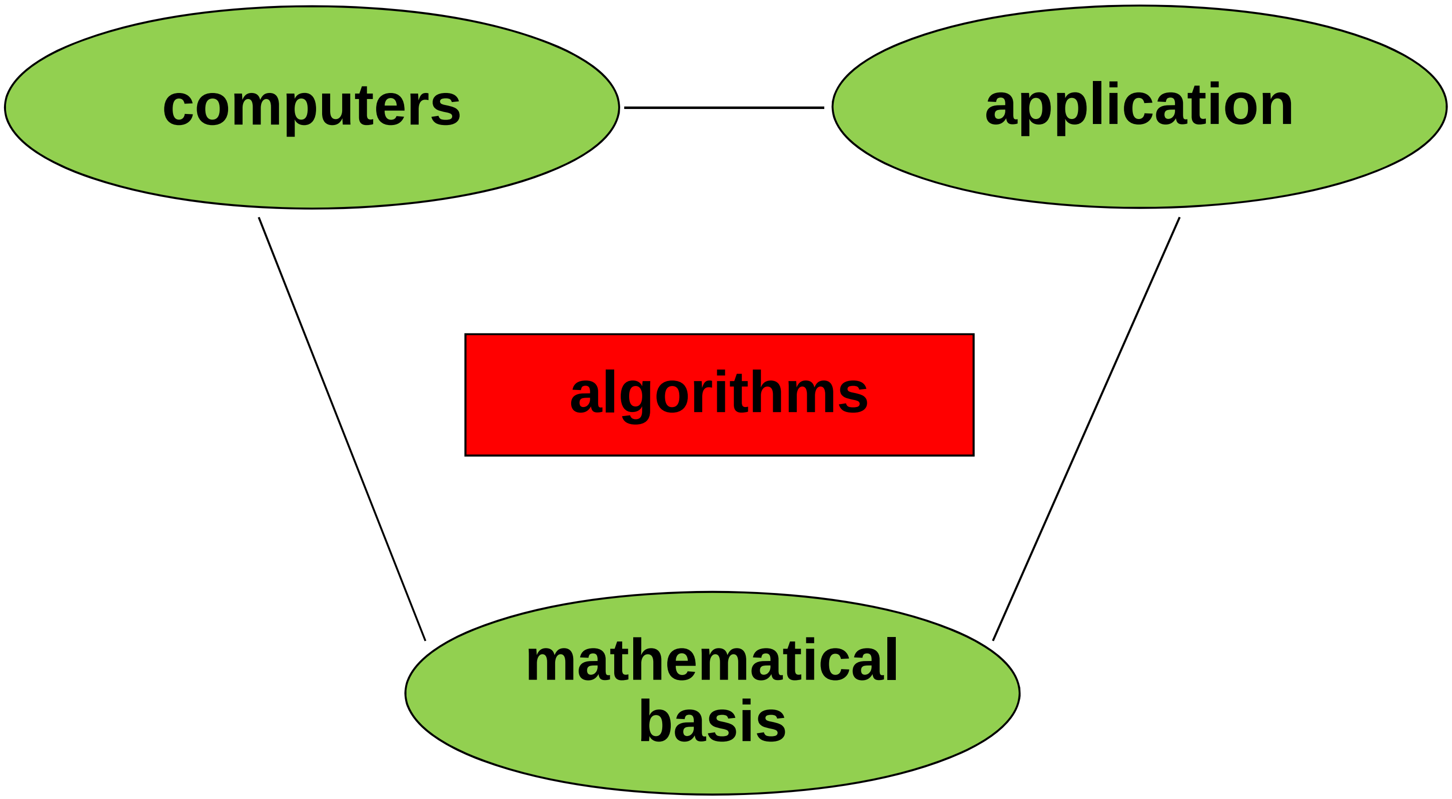
New role of mathematics



lateral thinking ⇔ transfer of ideas

Bring algorithms to school



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Algorithms in mathematical education

Algorithms (modules) are building blocks in
close to reality mathematical education

Modules tested and used in high schools:

- Multidisciplinary education
  combining mathematics and computer science
- Learn a simple programming language
  → e. g. Python
- Solve real problems from science and technology,
  in team work in different classes

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Today’s situation needs to be changed

Many teachers

- know little about the breakthroughs in Applied Mathematics
- don’t know much about (and don’t study) mathematical modeling, algorithms, numerics, differential equations, stochastics
- are not able to teach a simple programming language

Computers are used for surfing, mailing, skyping, playing games, facebooking but not for computing or problem solving

Today’s situation needs to be changed

- provide teaching material (modules)
- train teachers
- extend curricula
- renovate school books
- change (extend) teachers’ university studies

Multidisciplinary competence in mathematics, informatics, science and technology

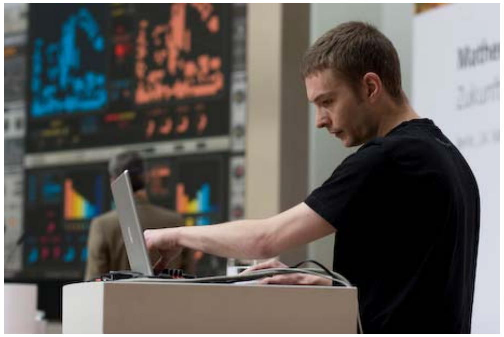
Approach:

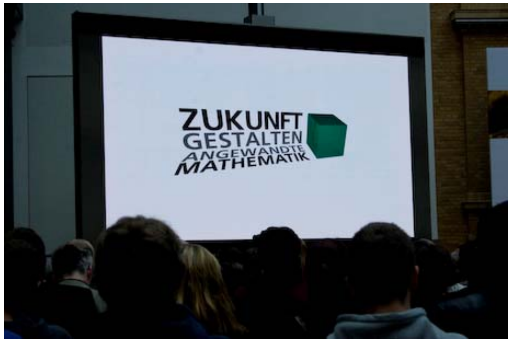
- deal with real problems (no fake- or pseudo-problems)
- discuss different mathematical models
- try to find your own algorithm
- write a computer program and test it (optionally)

Multidisciplinary competence in mathematics, informatics, science and technology

Students

- understand why mathematics is so important
- solve real problems
- enjoy inquiry-based work
- like to work with mathematical objects and structures





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